1.3 DO NOW:

1. The 4 aces are removed from a deck of cards. A coin is tossed and one of the aces is chosen. Describe the sample space.
2. Nelson has 3 cards and 2 chocolates. He chooses one card and one chocolate at random. How many outcomes are possible for which chocolate and card he picks?
3. A book shelf contains 6 history books, 9 mathematics books, and 4 accounting books. What is the probability of selecting a history book or accounting from the shelf?
4. Ray and Shan are playing football. Probability of Ray winning the football game is 0.36 what is the probability of Shan winning?
5. A number between 30 and 50 is chosen at random. What's the probability that the number contains at least one 4?

1.3 PROBABILITY OF INDEPENDENT AND DEPENDENT EVENTS

SWBAT:

- UNDERSTAND THAT TWO EVENTS A AND B ARE INDEPENDENT IF THE PROBABILITY OF A AND B OCCURRING TOGETHER IS THE PRODUCT OF THEIR PROBABILITIES, AND USE CHARACTERISTICS TO DETERMINE IF THEY ARE INDEPENDENT (S-CP.2)
- AND APPLY THE GENERAL MULTIPLICATION RULE IN A UNIFORM PROBABILITY MODEL, $P(A \text{ AND } B)=P(A)P(B|A)=P(B)P(A|B)$, AND INTERPRET THE ANSWER IN TERMS OF THE MODEL. (S-CP.8)

INDEPENDENT AND DEPENDENT EVENTS

- **Independent Events:** two events are said to be independent when one event has no affect on the probability of the other event occurring.
- **Dependent Events:** two events are dependent if the outcome or probability of the first event affects the outcome or probability of the second.

INDEPENDENT EVENTS

Suppose a die is rolled and then a coin is tossed.

- Explain why these events are independent.
  - They are independent because the outcome of rolling a die does not affect the outcome of tossing a coin, and vice versa.
- We can construct a table to describe the sample space and probabilities:

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head						
Tail						

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head	1,H	2,H	3,H	4,H	5,H	6,H
Tail	1,T	2,T	3,T	4,T	5,T	6,T

3. How many outcomes are there for rolling the die?
    - 6 outcomes
4. How many outcomes are there for tossing the coin?
    - 2 outcomes
5. How many outcomes are there in the sample space of rolling the die and tossing the coin?
    - 12 outcomes

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head	1,H	2,H	3,H	4,H	5,H	6,H
Tail	1,T	2,T	3,T	4,T	5,T	6,T

6. Is there another way to decide how many outcomes are in the sample space?

  - Multiply the number of outcomes in each event together to get the total number of outcomes.

- Let's see if this works for another situation.

A FAST FOOD RESTAURANT OFFERS 5 SANDWICHES AND 3 SIDES. HOW MANY DIFFERENT MEALS OF A SANDWICH AND SIDE CAN YOU ORDER?

7. If our theory holds true, how could we find the number of outcomes in the sample space?

- 5 sandwiches x 3 sides = 15 meals

8. Make a table to see if this is correct.

	Sand. 1	Sand. 2	Sand. 3	Sand. 4	Sand. 5
Side 1					
Side 2					
Side 3					

- Were we correct?

FUNDAMENTAL COUNTING PRINCIPLE

If an event has ***m*** possible outcomes and another independent event has ***n*** possible outcomes, then there are $\boldsymbol{m_* n}$ possible outcomes for the two events together.

Fundamental Counting Principle can be used determine the number of possible outcomes when there are two or more characteristics

EXAMPLES

- 9. A student is to roll a die and flip a coin. How many possible outcomes will there be?

  6 X 2 = 12 outcomes

- 10. For a college interview, Robert has to choose what to wear from the following: 4 slacks, 3 shirts, 2 shoes and 5 ties. How many possible outfits does he have to choose from?

  4 X 3 X 2 X 5 = 120 outfits

PROBABILITIES OF INDEPENDENT EVENTS

The probability of independent events is the probability of both occurring, denoted by P(A and B) or $P(A \cap B)$.

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head	1,H	2,H	3,H	4,H	5,H	6,H
Tail	1,T	2,T	3,T	4,T	5,T	6,T

Use the table to find the following probabilities:

11. P(rolling a 3)

    2/12 = 1/6

12. P(Tails)

    6/12 = ½

13. P(rolling a 3 AND getting tails)

    1/12

14. P(rolling an even)

    6/12 = ½

15. P(heads)

    6/12 = ½

16. P(rolling an even AND getting heads)

    3/12 or 1/4

MULTIPLICATION RULE OF PROBABILITY

- The probability of two independent events occurring can be found by the following formula:

$$P(A \cap B) = P(A) \times P(B)$$

EXAMPLES

18. At City High School, 30% of students have part-time jobs and 25% of students are on the honor roll. What is the probability that a student chosen at random has a part-time job and is on the honor roll? Write your answer in context.

    P(PT job and honor roll) = P(PT job) x P(honor roll) = .30 x .25 = .075

    There is a 7.5% probability that a student chosen at random will have a part-time job and be on the honor roll.

The following table represents data collected from a grade 12 class in DEF High School.

Plans after High School			
Gender	**University**	**Community College**	**Total**
Males	28	56	84
Females	43	37	80
Total	71	93	164

Suppose 1 student was chosen at **random** from the grade 12 class.

19. What is the probability that the student is female?

80/164 = 20/41 = 0.488 or 48.8%

20. What is the probability that the student is going to university?

71/164

Now suppose 2 people both randomly chose 1 student from the grade 12 class. Assume that it's possible for them to choose the same student.

21. What is the probability that the first person chooses a student who is female and the second person chooses a student who is going to university?

    (80/164) X (71/164) = (355/1681) = (71/336) or 0.211 or 21.1%

3. SUPPOSE A CARD IS CHOSEN AT RANDOM FROM A DECK OF CARDS, REPLACED, AND THEN A SECOND CARD IS CHOSEN.

22. Would these events be independent? How do we know?
    - Yes, because the first card is replaced before the second card is drawn.

23. What is the probability that both cards are 7s?
    - P(7) = 4/52, so P(7 and 7) = P(7) x P(7) = 4/52 x 4/52 = 1/169 or .0059.
    - This means that the probability of drawing a 7, replacing the card and then drawing another 7 is 0.59%

DEPENDENT EVENTS

- Remember, we said earlier that
    - **Dependent Events:** two events are dependent if the outcome or probability of the first event affects the outcome or probability of the second.
- Let's look at some scenarios and determine whether the events are independent or dependent.

DETERMINE WHETHER THE EVENTS ARE INDEPENDENT OR DEPENDENT:

24. Selecting a marble from a container and selecting a jack from a deck of cards.
    - Independent
25. Rolling a number less than 4 on a die and rolling a number that is even on a second die.
    - Independent
26. Choosing a jack from a deck of cards and choosing another jack, without replacement.
    - Dependent
27. Winning a hockey game and scoring a goal.
    - Dependent

PROBABILITIES OF DEPENDENT EVENTS

- We cannot use the multiplication rule for finding probabilities of dependent events because the one event affects the probability of the other event occurring.
- Instead, we need to think about how the occurrence of one event will effect the sample space of the second event to determine the probability of the second event occurring.
- Then we can multiply the new probabilities.

EXAMPLES

Suppose a card is chosen at random from a deck, the card is NOT replaced, and then a second card is chosen from the same deck. What is the probability that both will be 7s?

28. This is similar the earlier example, but these events are dependent? How do we know?
29. How does the first event affect the sample space of the second event?

Suppose a card is chosen at random from a deck, the card is NOT replaced, and then a second card is chosen from the same deck.

30. Calculate the probability that both will be 7s?

- Let's break down what is going on in this problem:
  - We want the probability that the first card is a 7, or P(1st is 7), and the probability that the second card is a 7, or P(2nd is 7).
  - P(1st is 7) = 4/52 because there a four 7s and 52 cards
  - How is P(2nd is 7) changed by the first card being a 7?
  - P(2nd is 7) = 3/51
  - P(1st is 7, 2nd is 7) = 4/52 x 3/51 = 1/221 or .0045
  - The probability of drawing two sevens without replacement is 0.45%

31. A box contains 5 red marbles and 5 purple marbles. What is the probability of drawing 2 purple marbles and 1 red marble in succession *without replacement*?

- P(1^{st} purple) = 5/10
- P(2^{nd} purple) = 4/9
- P(3^{rd} red) = 5/8
- P(purple,purple,red) = 5/10 x 4/9 x 5/8 = 5/36 or .139
- The probability of drawing a purple, a purple, then a red without replacement is 13.9%

32. In Example 31, what is the probability of first drawing all 5 red marbles in succession and then drawing all 5 purple marbles in succession *without replacement*?

- P(5 red then 5 purple) = (5/10)(4/9)(3/8)(2/7)(1/6)(5/5)(4/4)(3/3)(2/2)(1/1) = 1/252 or .004
- The probability of drawing 5 red then 5 purple without replacement is 0.4%
- Explain why the last 5 probabilities above were all equivalent to 1.
- This is because there were only purple marbles left, so the probability for drawing a purple marble was 1.

1.3 CYU