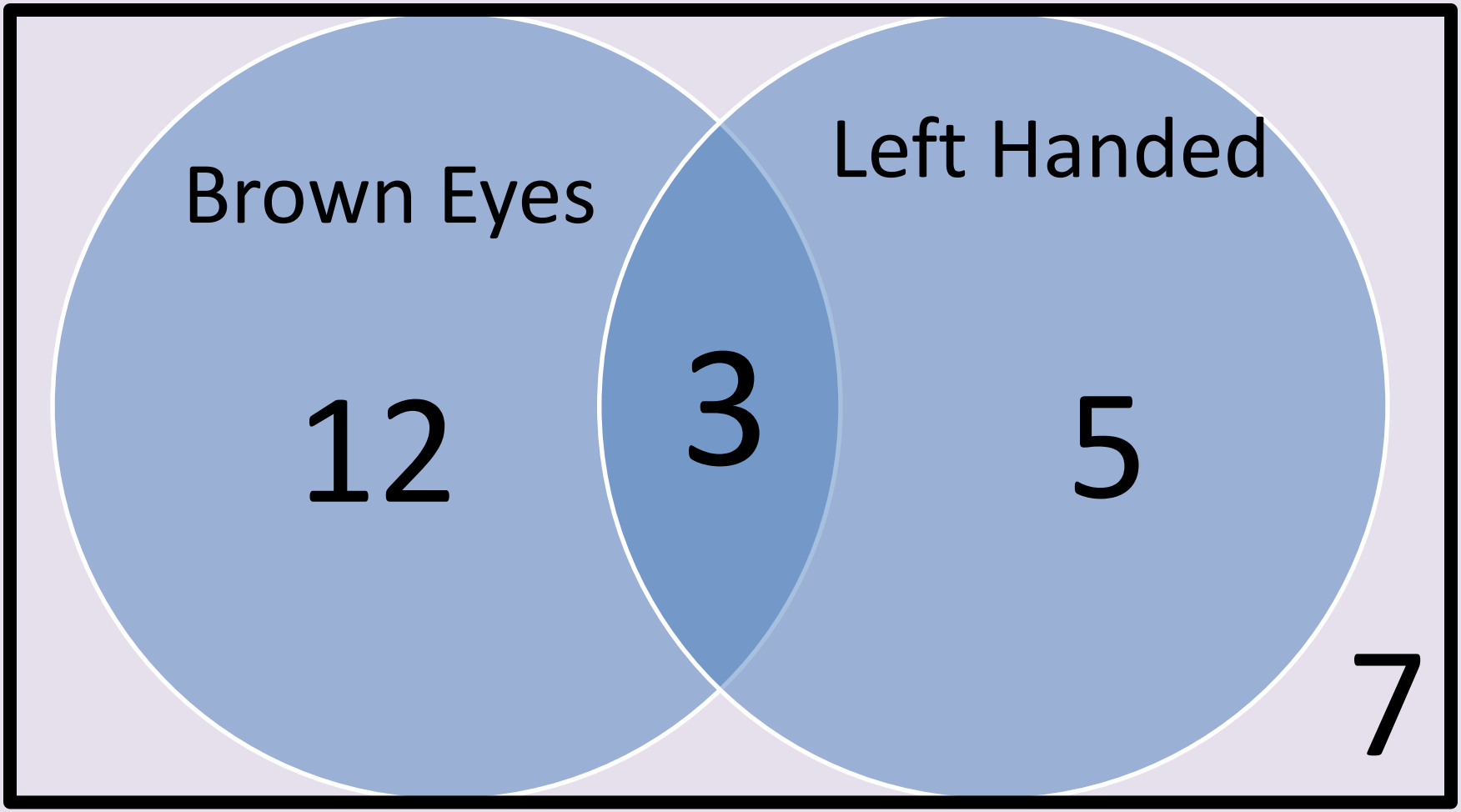
Do Now:

1. Explain how to determine if two events are independent or dependent.
2. Provide TWO examples of mutually exclusive events.
3. Provide TWO examples of mutually inclusive events.
4. In Ms. Plant's homeroom class, there are 27 students. 15 students who have brown eyes, 8 students who are left-handed and 7 students who have neither brown eyes nor are left-handed. Draw a Venn diagram and find how many of the students have both brown eyes and are left-handed.

Do Now #4



1.6 Conditional Probability

CCM2 Unit 1: Probability

Conditional Probability

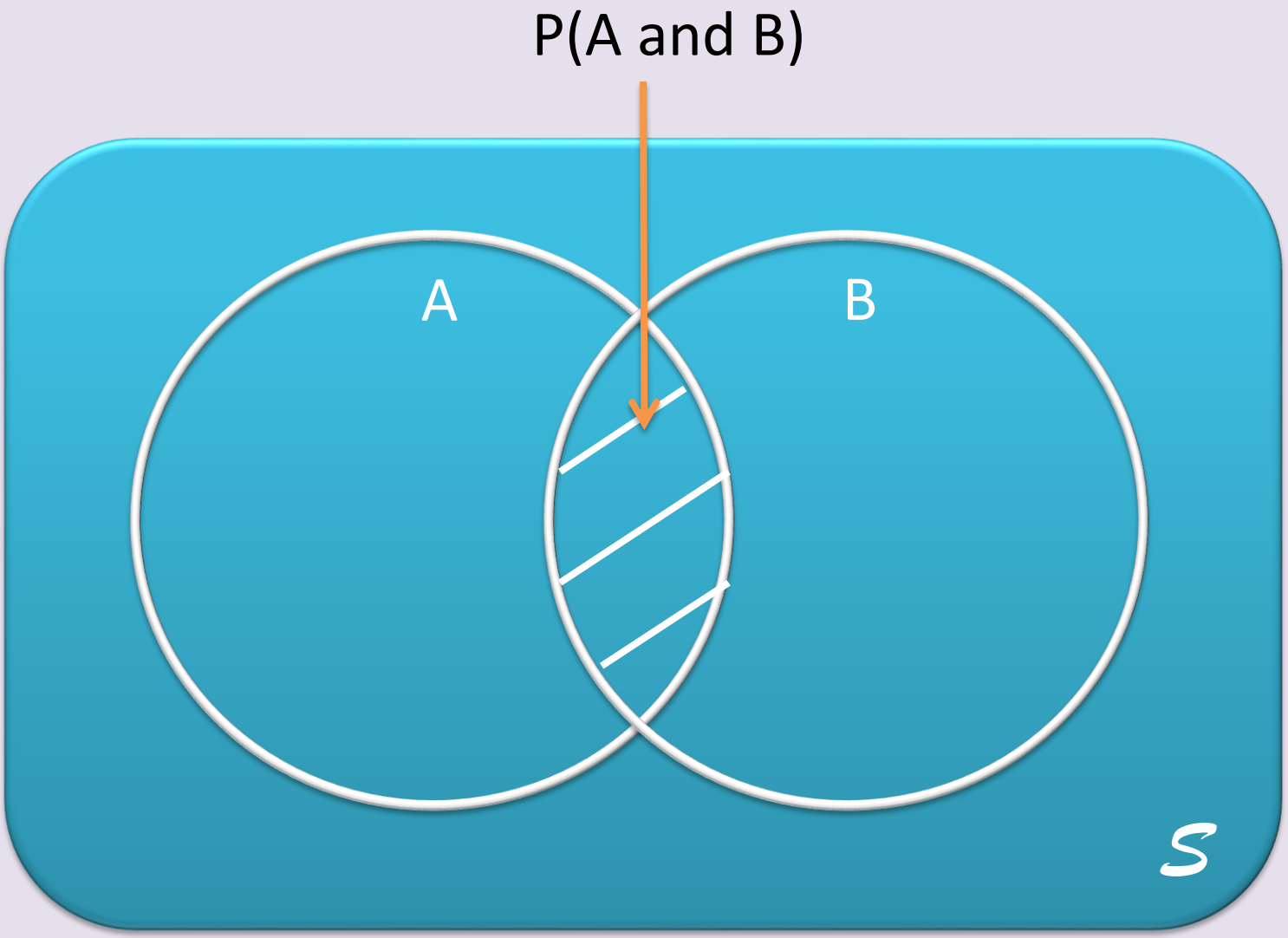
- **Conditional Probability:** A probability where a certain prerequisite condition has already been met.
- For example:
  - What is the probability of selecting a queen given an ace has been drawn and not replaced.
  - What is the probability that a student in the 10^{th} grade is enrolled in biology given that the student is enrolled in CCM2?

Conditional Probability Formula

- The conditional probability of A given B is expressed as P(A | B)

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

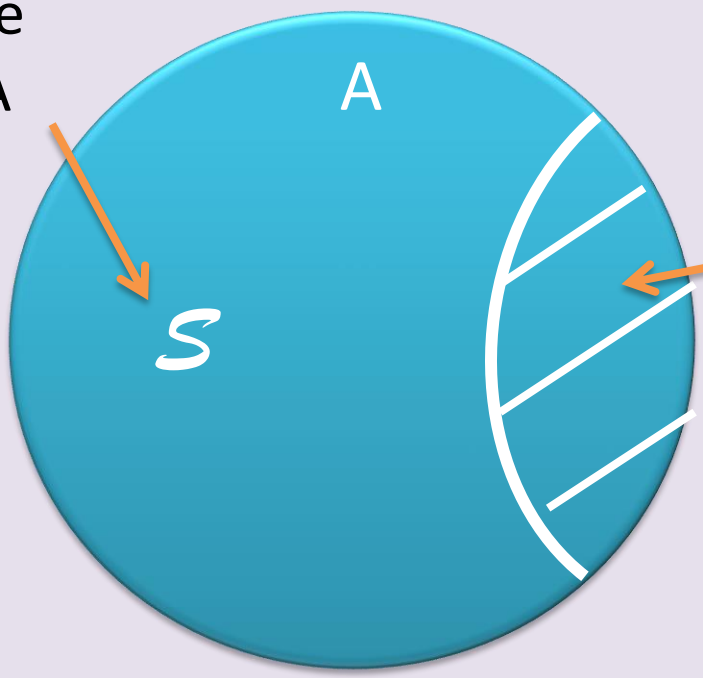
Joint Probability



Conditional Probability

Since Event A has happened, the sample space is reduced to the outcomes in A

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$



P(A and B) represents the outcomes from B that are included in A

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

$$P(2nd\ Club|1st\ Club) = \frac{P(1st\ Club\ and\ 2nd\ Club)}{P(1st\ Club)}$$

$$P(2nd\ Club|1st\ Club) = \frac{\frac{13}{52} * \frac{12}{51}}{\frac{13}{52}}$$

$$\frac{12}{51}\ or\ \frac{4}{17}\ or\ 23.5\%$$

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

$$P(Brown|Blue) = \frac{P(Brown\ and\ Blue)}{P(Blue)}$$

$$P(Brown|Blue) = \frac{\frac{6}{8} * \frac{2}{7}}{\frac{6}{8}}$$

$$\frac{2}{7}\ or\ 28.6\%$$

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

$$P(Brown|Blue) = \frac{P(1st\ Club\ and\ 2nd\ Club)}{P(1st\ Club)}$$

$$P(2nd\ Club|1st\ Club) = \frac{\frac{13}{52} * \frac{12}{51}}{\frac{13}{52}}$$

$$\frac{12}{51}\ or\ \frac{4}{17}\ or\ 23.5\%$$

Using Two-Way Frequency Tables to Compute Conditional Probabilities

- Let’s look at some examples to review this.

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	
11th or 12th	41	58	184	7	
Total					

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

Suppose we randomly select one student.

4. What is the probability that the student walked to school?

$$\frac{88}{500} \; or \; 17.6\%$$

5. P(9th or 10th grader)

$$\frac{210}{500} \; or \; 42\%$$

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

6. P(rode the bus OR 11th or 12th grader)

$$\frac{147}{500} + \frac{290}{500} - \frac{41}{500} = \frac{396}{500} \; or \; 79.2\%$$

	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	210
11^{th} or 12^{th}	41	58	184	7	290
Total	147	88	254	11	500

7. What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

$$P(11^{th} \text{ or } 12^{th} | \text{car})$$

- We only want to look at the car column for this probability!

$$\frac{11th\ or\ 12\ graders\ \cap\ in\ cars}{Total\ in\ cars}$$

$$\frac{184}{254}\ or\ 72.4\%$$

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

8. What is P(Walk|9th or 10th grade)?

$$\frac{Walk \cap 9th\ or\ 10th\ graders}{Total\ 9th\ and\ 10th\ graders}$$

$$\frac{30}{210} = \frac{1}{7} = 14.2\%$$

The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

	Vanilla	Chocolate	Total
Adult			
Child			
Total			

	Vanilla	Chocolate	Total
Adult	52		93
Child			
Total		146	224

	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

9. Find P(vanilla|adult)

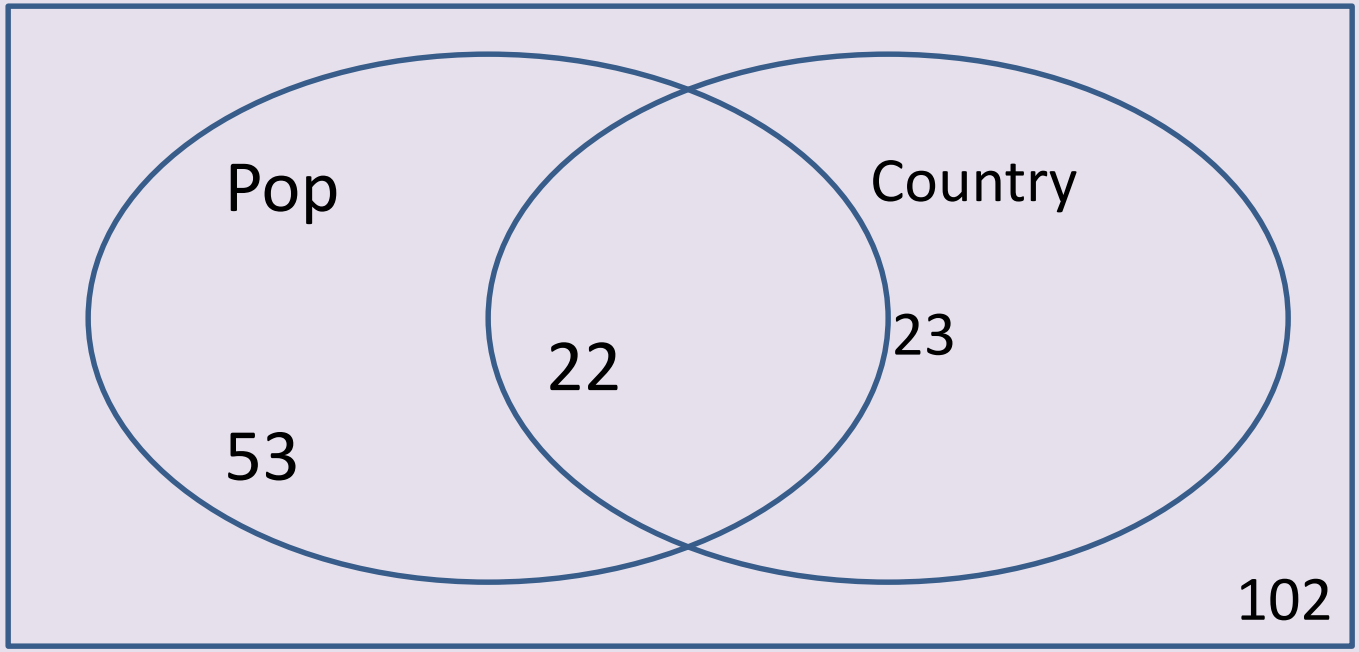
$$\frac{52}{90} \; or \; 55.9\%$$

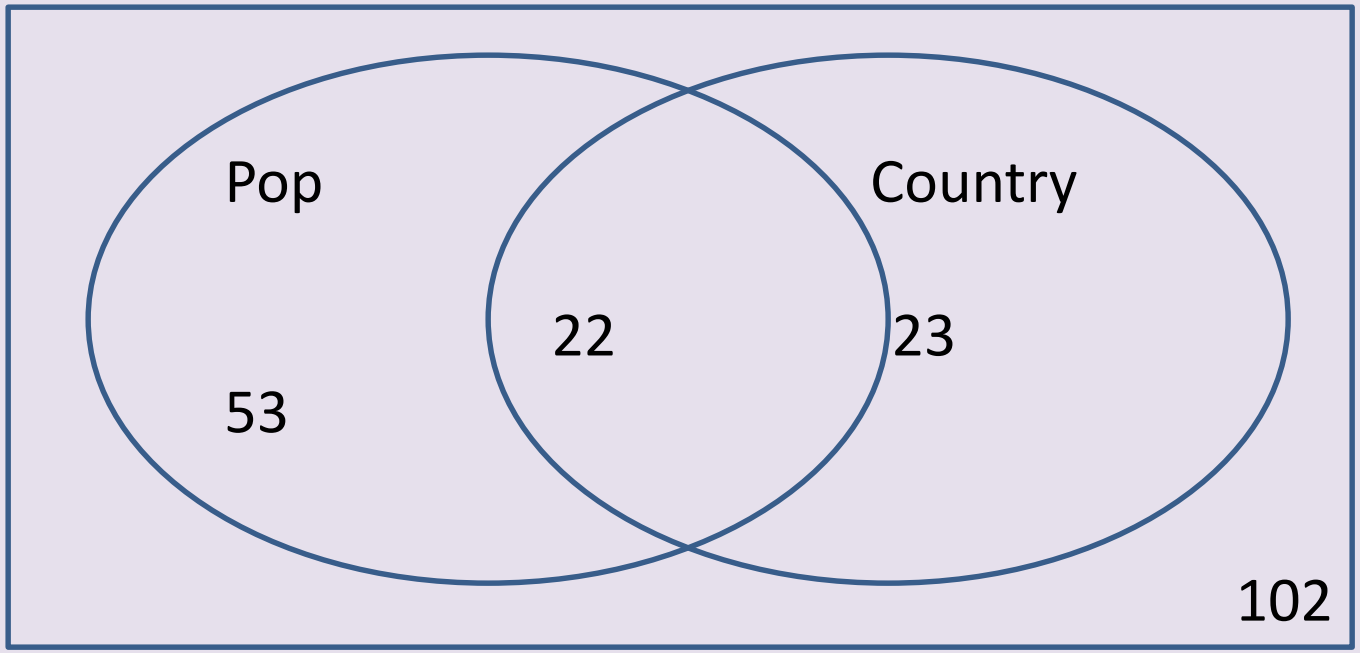
	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

10. Find P(child|chocolate)

$$\frac{105}{146} \; or \; 71.9\%$$

3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.





P(Pop|Country)

$$\frac{Pop \cap Country}{Country} = \frac{22}{22+23}$$

$$\frac{22}{45} \; or \; 48.9\%$$

Check Your Understanding (CYU)

Complete the conditional probability worksheet and turn in to be checked