

To the right is a graph of a "piece-wise" function. We'll call this function  $F(x)$ . We can use  $F(x)$  to explore transformations in the coordinate plane.

1. How do we know that  $F(x)$  is a function?  
*(Hint: How do we define a function?)*

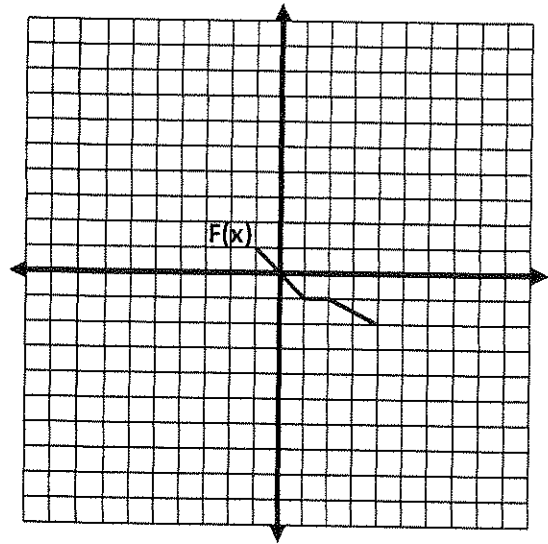
for every  $x$  value, there is one  $y$  value

2. What is the domain of  $F(x)$ ?

$$\{-1 \leq x \leq 4\}$$

3. What is the range of  $F(x)$ ?

$$\{-2 \leq y \leq 1\}$$



Let's explore the points on  $F(x)$ .

4. How many points lie on  $F(x)$ ? Can we list them all?

there is an infinite number of points

5. What are the key points that would help us graph  $F(x)$ ?

We will call these key points "characteristic" points. It is important when graphing a function that you are able to identify these characteristic points.

- beginning
- end
- turn

6. Use the graph of  $F(x)$  to evaluate the following:

$$F(1) = -1 \quad F(-1) = 1 \quad F(5) = \text{undefined}$$

(it's not on the line)

\*Remember that  $F(x)$  is another name for the  $y$ -values\*

7. Fill the three tables using the graph of  $F(x)$ .

x	$F(x)$
-1	1
1	-1
2	-1
4	-2

x	$F(x) + 4$
-1	5
1	3
2	3
4	2

x	$F(x) - 3$
-1	-2
1	-4
2	-4
4	-5

8. Graph  $F(x) + 4$  and  $F(x) - 3$  in different colors on the coordinate plane above

(Use the coordinate plane above)

9. In  $y = F(x) + 4$ , how did the "+4" affect the graph of  $F(x)$ ? What type of transformation maps  $F(x)$  to  $F(x) + 4$ ? (Be specific)

it translated up 4

10. In  $y = F(x) - 3$ , how did the "-3" affect the graph of  $F(x)$ ? What type of transformation maps  $F(x)$  to  $F(x) - 3$ ? (Be specific)

it translated down 3

11. Fill the three tables using the graph of  $F(x)$ .

x	F(x)
-1	1
1	-1
2	-1
4	-2

x	x + 4	y = F(x + 4)
-5	-1	1
-3	1	-1
-2	2	-1
0	4	-2

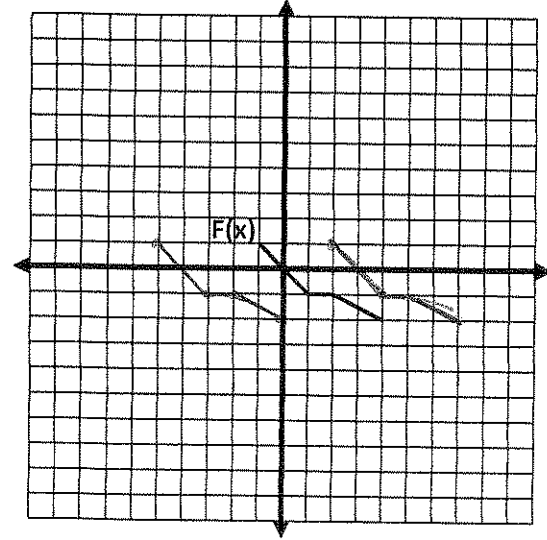
Hint: In the first box we have  $x + 4 = -1$ . If we subtract 4 from both sides of the equation, we get  $x = -5$ . Use a similar method to find the remaining  $x$  values.

x	(x - 3)	y = F(x - 3)
2	-1	1
4	1	-1
5	2	-1
7	4	-2

$f(x) + n$  = moves up by  $n$   
 $f(x) - n$  = moves down by  $n$   
 $f(x + n)$  = moves left by  $n$   
 $f(x - n)$  = moves right by  $n$

12. On the coordinate plane to the right:

- \* a. Use one color to graph the 4 ordered pairs  $(x, y)$  for  $y = F(x + 4)$ . The first point is  $(-5, 1)$ .  
*red*
- \* b. Use a different color to graph the 4 ordered pairs  $(x, y)$  for  $y = F(x - 3)$ .  
*blue*



13. In  $y = F(x + 4)$ , how did the "+4" affect the graph of  $F(x)$ ? What type of transformation maps  $F(x)$  to  $F(x + 4)$ ? (Be specific)

translated to the left by 4

14. In  $y = F(x - 3)$ , how did the "-3" affect the graph of  $F(x)$ ? What type of transformation maps  $F(x)$  to  $F(x - 3)$ ? (Be specific)

translated to the right by 3

15. Fill the tables using the graph of  $F(x)$ .

x	F(x)
-1	1
1	-1
2	-1
4	-2

x	-F(x)
-1	-1
1	1
2	1
4	2

x	2F(x)
-1	2
1	-2
2	-2
4	-4

x	1/2 F(x)
-1	1/2
1	-1/2
2	-1/2
4	-1

\*when you multiply a function by a negative number, it will reflect across the x-axis

vertical stretch

vertical shrink

16. How did each of the following affect the graph of $F(x)$ :	<i>Hint: Use one of the coordinate planes above if needed.</i>
a) the "-" sign	a) reflect over the $x$ -axis
b) the "2"	b) vertical stretch
c) the " $\frac{1}{2}$ "	c) vertical shrink

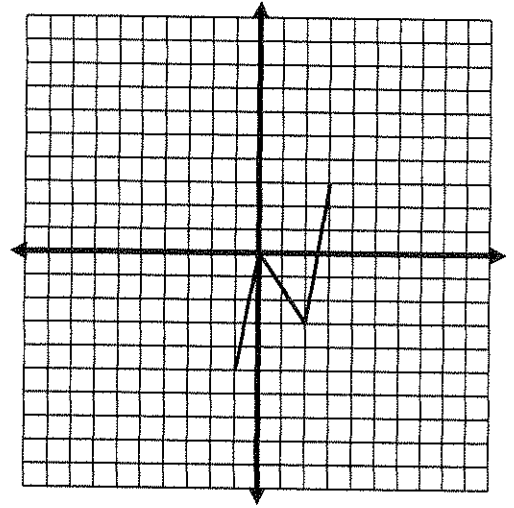
Summary: Describe the effect to  $F(x)$  for the following functions.

Equation	Effect on the graph of $F(x)$
Example: $y = F(x + 18)$	Translate $F(x)$ to the left 18 units
1. $y = F(x) - 100$	down 100
2. $y = F(x - 48)$	right 48
3. $y = F(x) + 32$	up 32
4. $y = -F(x)$	reflect over $x$ -axis
5. $y = F(x - 10)$	right 10
6. $y = F(x) + 7$	up 7
7. $y = \frac{1}{4}F(-x)$	vertical shrink by $\frac{1}{4}$ , reflect over $y$ -axis
8. $y = F(x) - 521$	down 521
9. $y = F(x) + 73$	up 73
10. $y = -5F(x)$	vertical stretch by 5, reflect over $x$ -axis
11. $y = F(x) - 22$	down 22
12. $y = 2F(x - 13)$	vertical stretch by 2, right 13
13. $y = F(x + 30) + 18$	left 30, up 18
14. $y = -\frac{1}{4}F(\frac{1}{3}x) - 27$	vertical shrink $\frac{1}{4}$ , reflect over the $x$ -axis, horizontal <u>stretch</u> of <u>3</u> , down <u>27</u>

take the reciprocal

To the right is a graph of a "piece-wise" function that we'll call  $H(x)$ .

Use  $H(x)$  to demonstrate what you have learned so far about the transformations of functions.



1. What are the characteristic points of  $H(x)$ ?

$(-1, -5)$   $(0, 0)$   $(2, -3)$   $(3, 3)$

2. Describe the effect on the graph of  $H(x)$  for each of the following:

a.  $H(x - 2)$  right 2

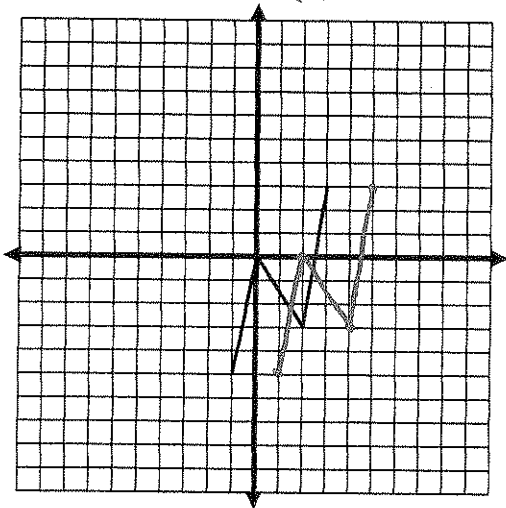
b.  $H(x) + 7$  up 7

c.  $H(x+2) - 3$  left 2, down 3

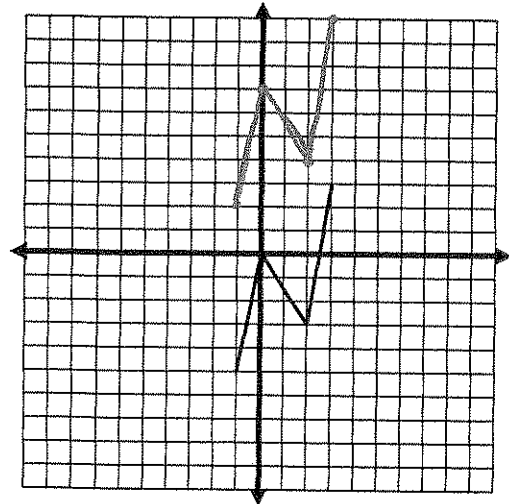
d.  $-2H(x)$  vertical stretch 2, reflect over x-axis

3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

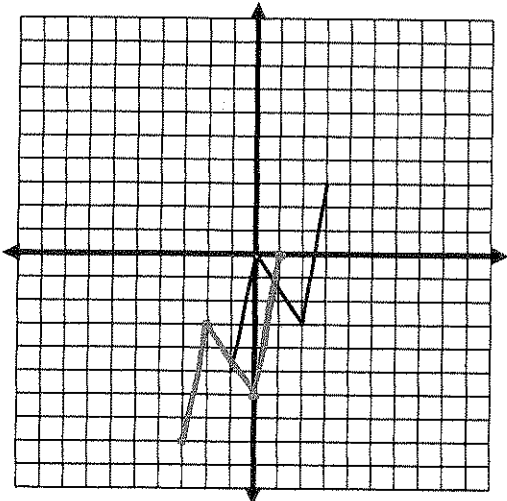
a.  $y = H(x - 2)$  (right 2)



b.  $y = H(x) + 7$  (up 7)



c.  $y = H(x+2) - 3$  (left 2, down 3)



d.  $y = -2H(x)$  (vertical stretch by 2, reflect over x-axis)

