Graphing Square and Cube Root Functions

Make a table for each function.

$f(x) = x^2$

x	f(x)
0	0
1	1
2	4
3	9
4	16

$f(x) = \sqrt{x}$

x	f(x)
0	0
1	1
4	2
9	3

$f(x) = x^3$

x	f(x)
-2	-8
-1	-1
0	0
1	1
2	8

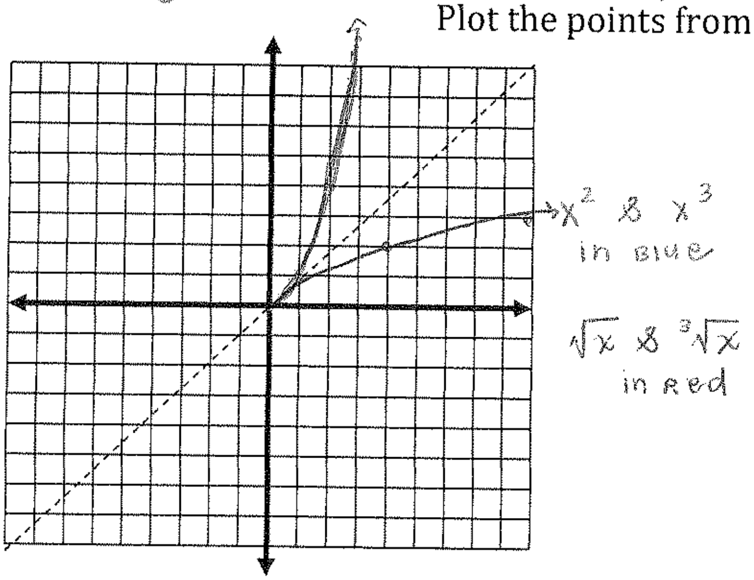
$f(x) = \sqrt[3]{x}$

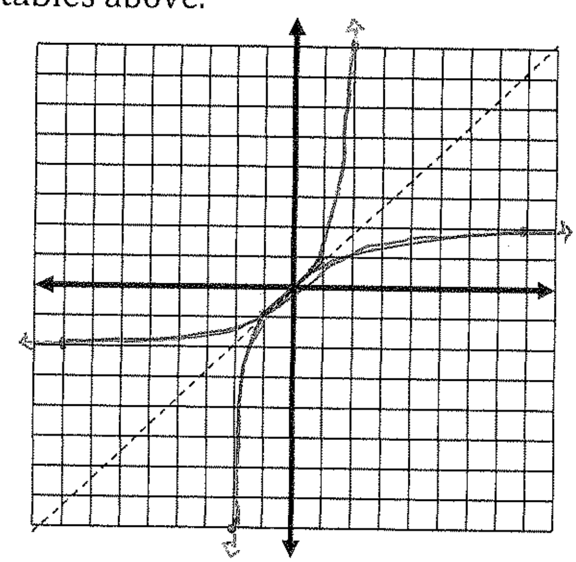
x	f(x)
-8	-2
-1	-1
0	0
1	1
8	2

Ignore the points with decimals. What do you notice about the other points?
the x and y values flip between each pair

These functions are inverses of each other. By definition, this means the x-values and the y-values invert (switch places)

Plot the points from the tables above.

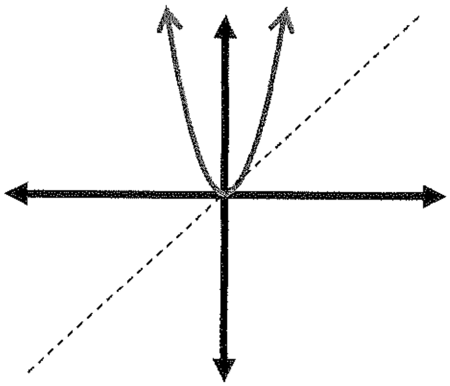


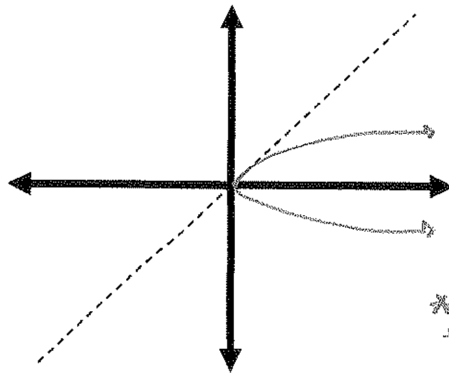


As a result, the graphs have the same numbers in their points but the x and the y coordinates have traded places.
This causes the graphs to have the same shape but to be reflected over the line y = x.

The Square and Square Root Function

Reflect the function $f(x) = x^2$ over the line $y = x$.



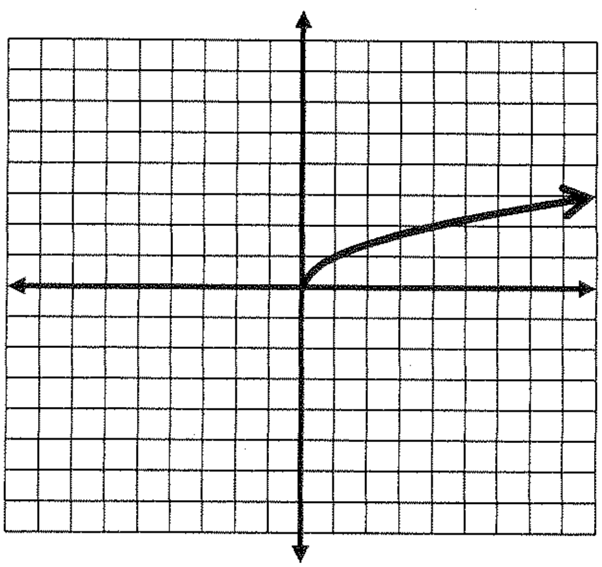


* one x-value has two y-values

Problems? It's NOT a function!

We have to define the Square Root graph as function. This means that we will only use the POSITIVE side of the graph.

The result: $f(x) = \sqrt{x}$



Characteristics of the graph

Vertex
(0, 0) * the starting point is the vertex

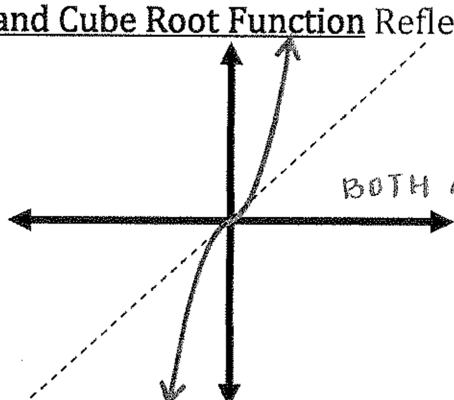
Domain
$x \geq 0$ * x can be 0, but it cannot be negative

Range
$y \geq 0$

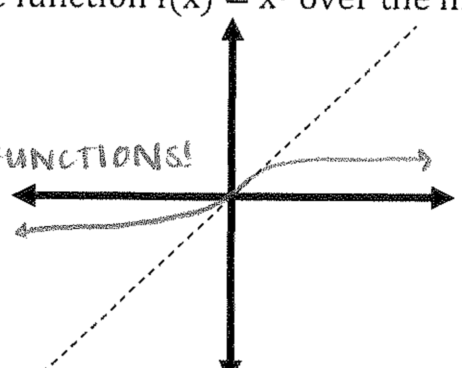
Symmetry
none

Pattern
square root of all values

The Cube and Cube Root Function Reflect the function $f(x) = x^3$ over the line $y = x$.



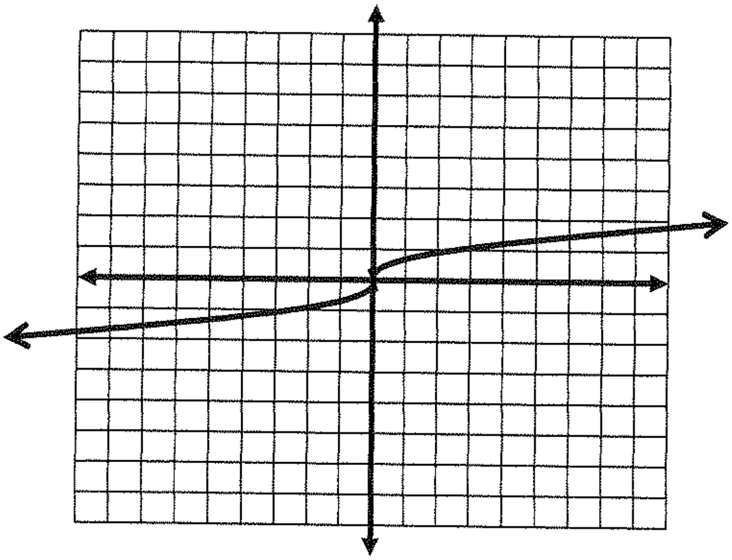
BOTH ARE FUNCTIONS!



Problems? none

The result: $f(x) = \sqrt[3]{x}$

Characteristics of the graph



Vertex
(0, 0)
Domain
all reals
Range
all reals
Symmetry
origin $(x, y) \to (-x, -y)$
Pattern
cube root of each x-value

Transforming the Graphs

Now that we know the shapes we can use what we know about transformations to put that shape on the coordinate plane. $y = a\sqrt{x-h}+k$ $y = a\sqrt[3]{x-h}+k$

Remember:

<u>Translate</u>
-h = to the right
+h = to the left
-k = down
+k = up

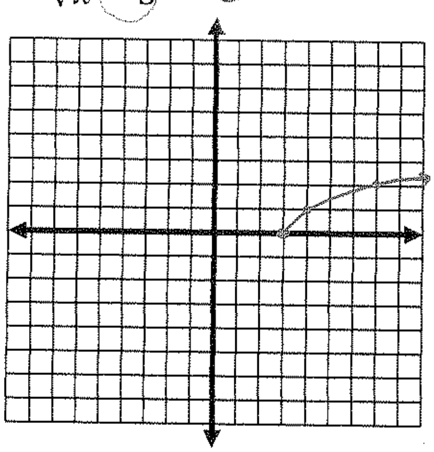
<u>Reflect</u>
$a < 0$ = reflect over the x-axis

<u>Dilate</u>
$a > 1$ = vertical stretch
$0 < a < 1$ = vertical shrink

1) $f(x) = \sqrt{x-3}$ right 3

$x = 4$
$\sqrt{4-3}$
$\sqrt{1}$
1
(4, 1)

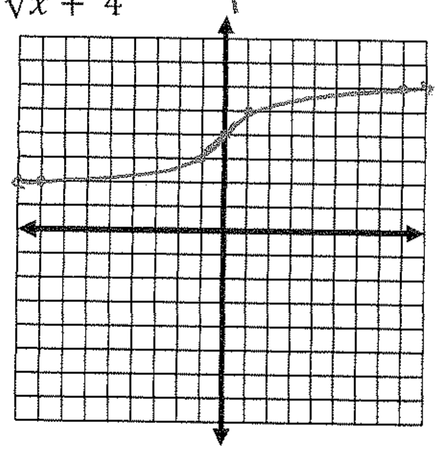
$x = 7$
$\sqrt{7-3}$ (7, 2)
$\sqrt{4}$
= 2



vertex = (3, 0)
Domain = $x \geq 3$ *was moved right, domain changes
Range = $y \geq 0$ *was not moved up/down, does not change

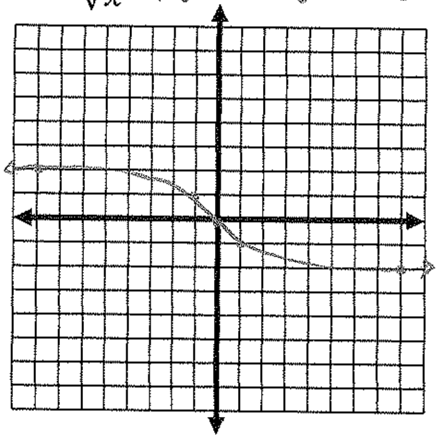
2) $f(x) = \sqrt[3]{x} + 4$ up 4

*cube root → you want perfect cube under radical
ex: -8, -1, 0, 1, 8



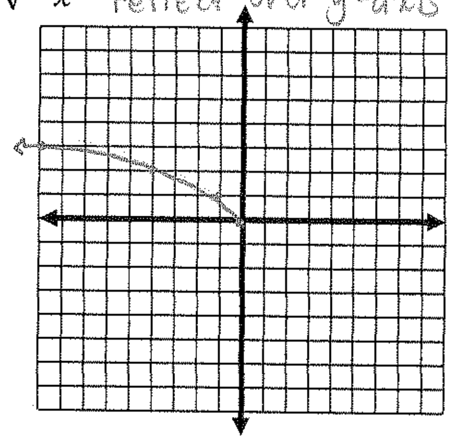
vertex = (0, 4)
Domain = all reals
Range = all reals

3) $f(x) = -\sqrt[3]{x}$ reflect over x



v = (0, 0)
D = all reals
R = all reals

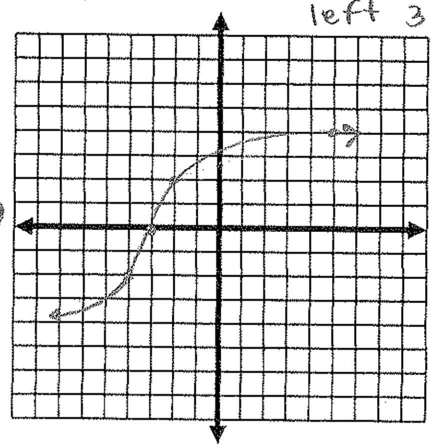
4) $f(x) = \sqrt{-x}$ reflect over y-axis



5) $f(x) = 2\sqrt[3]{x+3}$ vertical stretch 2, left 3

$x = -4$
$2\sqrt[3]{-4+3}$
$= 2\sqrt[3]{-1}$
$= 2(-1)$
$= -2$ $(-4, -2)$

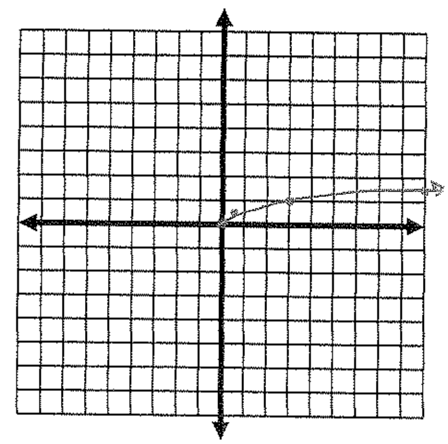
$x = 5$
$= 2\sqrt[3]{5+3}$
$= 2\sqrt[3]{8}$
$= 2(2)$ $(5, 4)$
$= 4$



vertex = (−3, 0)

6) $f(x) = \frac{1}{2}\sqrt{x}$ vertical shrink ½

vertex = (0, 0)



Sometimes the functions are not in graphing form. We may have to use some of our algebra skills to transform the equations into something we can use.

Ex: $f(x) = \sqrt{4x - 12}$ **This is not in graphing form.**

$= \sqrt{4(x-3)}$ → OR → $\sqrt{4} \cdot \sqrt{x-3} = 2\sqrt{x-3}$

$= [4(x-3)]^{1/2}$

$= 4^{1/2}(x-3)^{1/2}$

$= 2\sqrt{x-3}$

TRANSFORMATIONS
- vertical stretch 2
- right 3

vertex = (3, 0)

Ex: $f(x) = \sqrt[3]{8x + 32} - 5$ **This is not in graphing form.**

$\sqrt[3]{8(x+4)} - 5$

$(\sqrt[3]{8} \cdot \sqrt[3]{x+4}) - 5$

$2 \cdot \sqrt[3]{x+4} - 5$

$2\sqrt[3]{x+4} - 5$

vertex = (−4, −5)

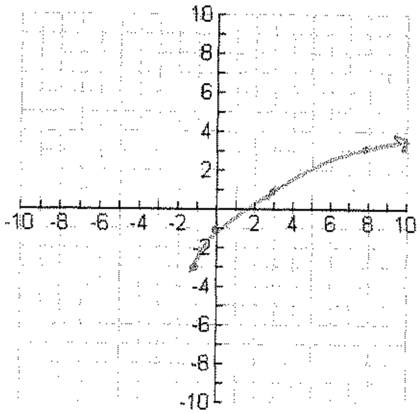
TRANSFORMATIONS
- vertical stretch 2
- left 4
- down 5

Graphing Radical Functions Practice

Graph each function, and identify its domain and range. For #5-6 put in graphing form first.

1. $f(x)=2\sqrt{x+1}-3$

vertex = (-1, -3)



Transformation: vertical stretch 2, left 1, down 3

Domain:

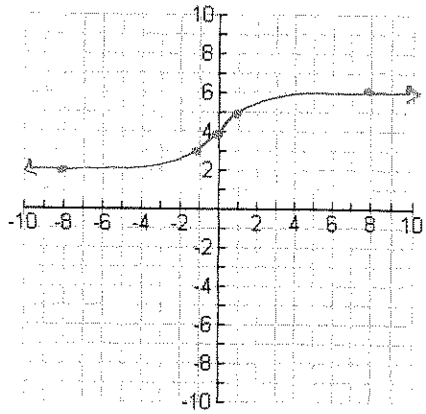
$x \geq -1$

Range:

$y \geq -3$

2. $f(x)=\sqrt[3]{x}+4$

vertex = (0, 4)



Transformation:

up 4

Domain:

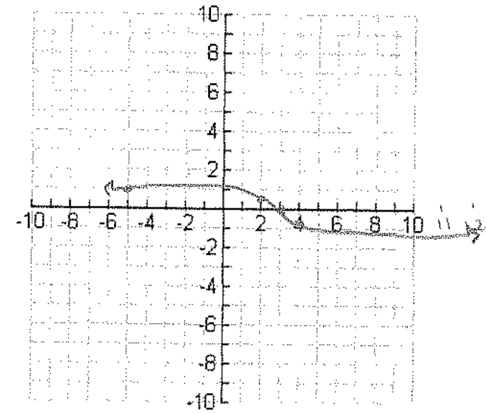
all reals

Range:

all reals

3. $f(x)=-\frac{1}{2}\sqrt[3]{x-3}$

vertex = (3, 0)



Transformation: right 3, reflect over x, shrink 1/2

Domain:

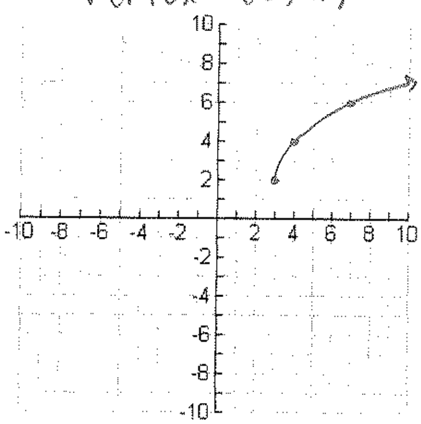
all reals

Range:

all reals

4. $f(x)=2\sqrt{x-3}+2$

vertex = (3, 2)



Transformation: right 3, up 2, vertical stretch 2

Domain:

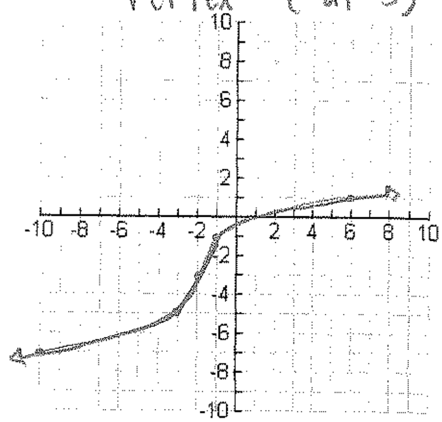
$x \geq 3$

Range:

$y \geq 2$

5. $f(x)=\sqrt[3]{8x+16}-3$

vertex = (-2, -3)



Graphing Form:

$2\sqrt[3]{x+2}-3$

Transformation: left 2, down 3, vertical stretch 2

Domain:

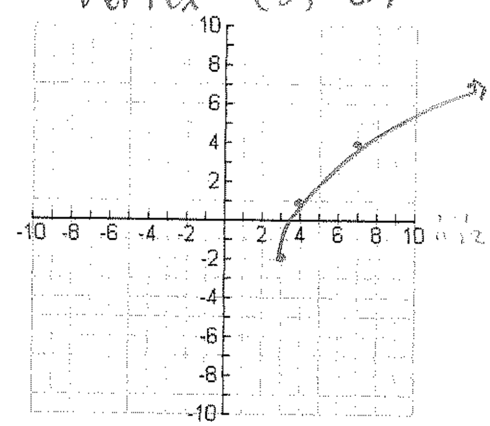
all reals

Range:

all reals

6. $f(x)=\sqrt{9x-27}-2$

vertex = (3, -2)



Graphing Form:

$3\sqrt{x-3}-2$

Transformation: right 3, down 2, vertical stretch 3

Domain:

$x \geq 3$

Range:

$x \geq -2$