Graphing Inverse Variation

A relationship that can be written in the form $\boxed{y = \frac{k}{x}}$, where k is a nonzero constant and $x \neq 0$, is an inverse variation.

parent function

The constant k is the constant of variation

Inverse variation implies that one quantity will increase while the other quantity will decrease (the inverse, or opposite, of increase).

The domain is all real numbers except zero. $x \neq 0$

Why?

you cannot divide a number by zero. since x is the denominator, it cannot be zero.

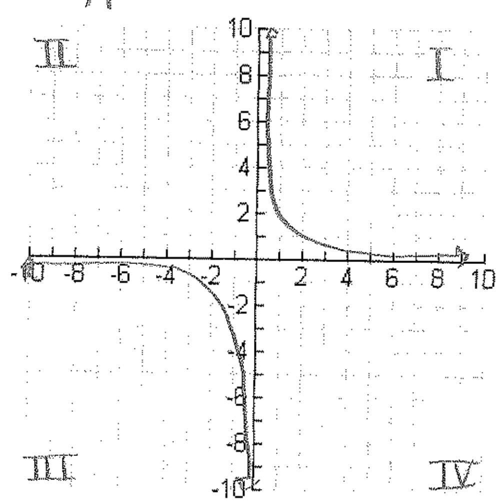
The range is all real numbers except zero. $y \neq 0$

Why? since k cannot be zero and x cannot be zero, there is NO WAY that y will ever be zero

Since both the domain and range have restrictions at zero, the graph can never touch the x and y axis.

This creates asymptotes at the axis.

"hyperbola"



The graphs of inverse variations have two parts.

Ex: $f(x) = \frac{1}{x}$

Each part is called a branch.

When k is positive, the branches are in Quadrants 1 and 3.

When k is negative, the branches are in Quadrants 2 and 4.

Translations of Inverse Variations:

The graph of y= $\frac{k}{x-b} + c$

is a translation of $y = k/x$, b units horizontally and c units vertically.

The vertical asymptote is $x = b$. The horizontal asymptote is $y = c$.

k tells us how far the branches have been STRETCHED from the asymptotes. We can use it to help us find out corner points to start our branches.

$\sqrt{k}$ is the distance from the asymptote.

Example 1: $y = \frac{1}{x - 3} + 4$

Vertical Asymptote:

$x = 3$

Horizontal Asymptote:

$y = 4$

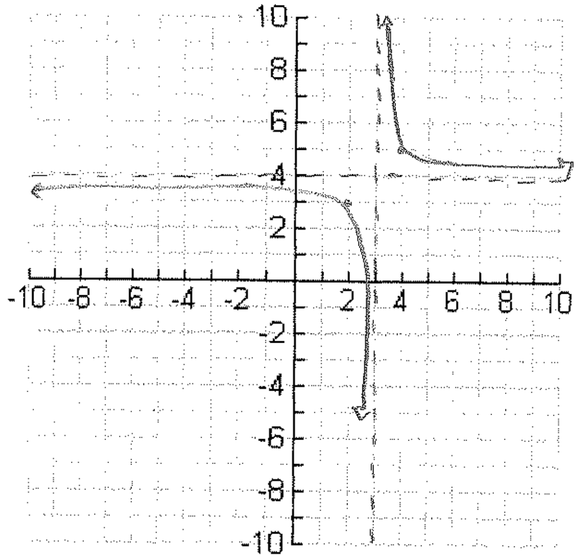
Quadrants:

1 & 3

Distance from the Asymptote:

$= \sqrt{k}$

$= \sqrt{1}$ $= 1$



* draw new asymptotes

* Distance: go up one, right one and down one, left one & place a point

You Try: $y = -\frac{4}{x + 1}$

Vertical Asymptote:

$x = -1$

Horizontal Asymptote:

$y = 0$

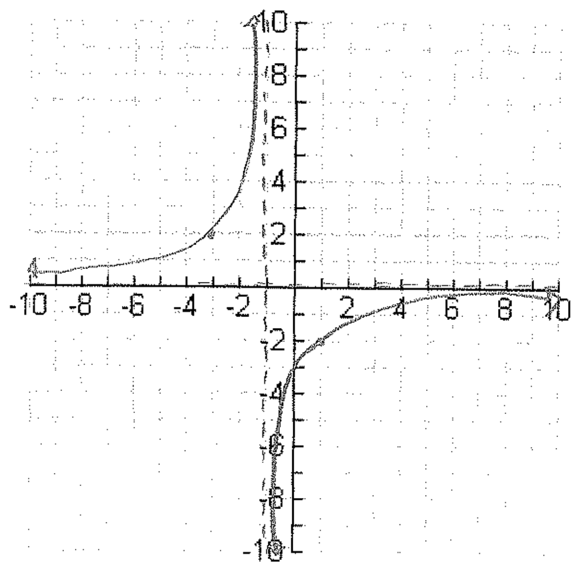
Quadrants:

2 & 4 * because of the negative

Distance from the Asymptote:

$= \sqrt{k}$

$= \sqrt{4}$ $= 2$



We can also write the equation just given the parent function and the asymptotes.

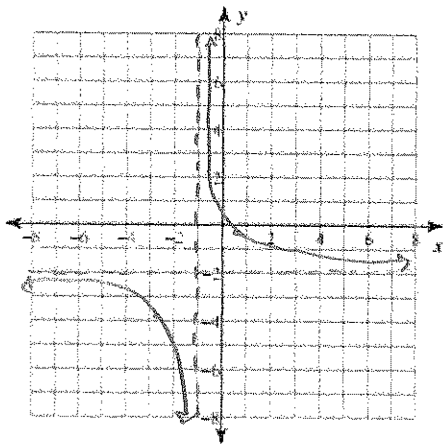
Example: Write the equation of $y = -\frac{1}{x}$ that has asymptotes x = -4 and y = 5.

$y = -\frac{1}{x + 4} + 5$

Graphing Inverse Variation Practice

For each of the graphs, identify the horizontal and vertical asymptotes, quadrants where it is located and the distance from the asymptotes.

1. $f(x) = \dfrac{3}{x+1} - 2$



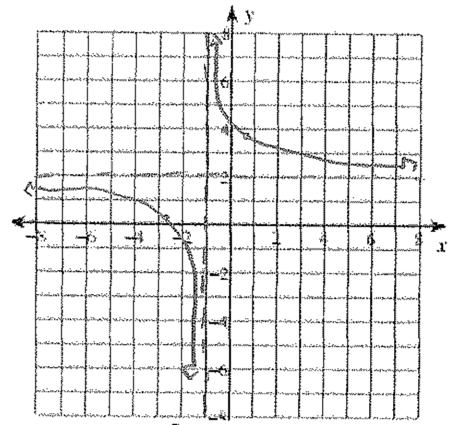
V.A. $x = -1$

H.A. $y = -2$

Quads 1 & 3

Dist. 1.7

2. $f(x) = \dfrac{3}{x+1} + 2$



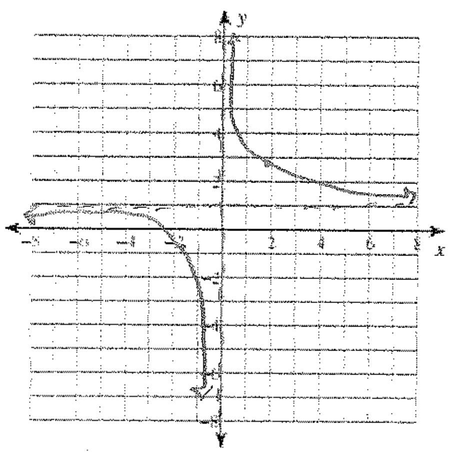
V.A. $x = -1$

H.A. $y = 2$

Quads 1 & 3

Dist. 1.7

3. $f(x) = \dfrac{3}{x} + 1$



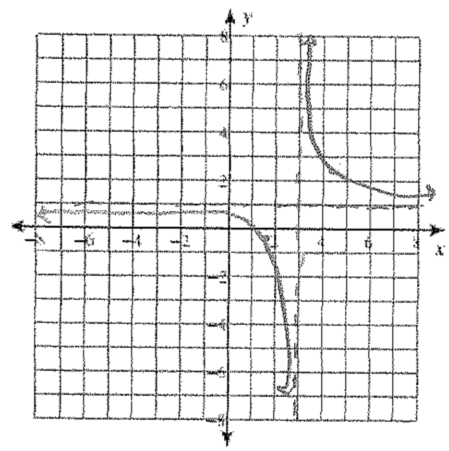
V.A. $x = 0$

H.A. $y = 1$

Quads 1 & 3

Dist. 1.7

4. $f(x) = \dfrac{2}{x-3} + 1$



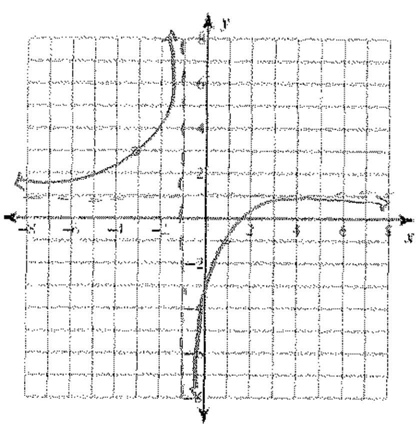
V.A. $x = 3$

H.A. $y = 1$

Quads 1 & 3

Dist. 1.4

5. $f(x) = -\dfrac{4}{x+1} + 1$



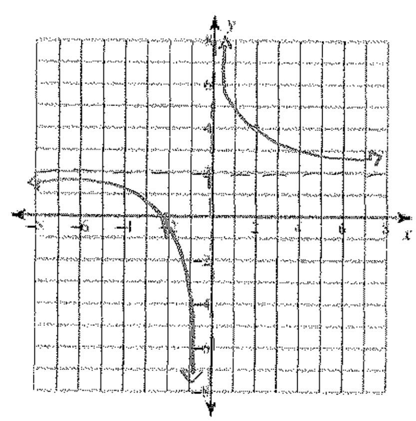
V.A. $x = -1$

H.A. $y = 1$

Quads 2 & 4

Dist. 2

6. $f(x) = \dfrac{4}{x} + 2$



V.A. $x = 0$

H.A. $y = 2$

Quads 1 & 3

Dist. 2

7. Write the equation of $y = -\frac{1}{x}$ that has asymptotes x = 2 and y = 4 that is 3 units from the asymptotes.

$y = -\dfrac{9}{x-2} + 4$