Exponent Rules:

Product of powers: $x^m * x^n = x^{m+n}$

Quotient of powers: $\frac{x^m}{x^n} = x^{m-n}$

Negative exponents: $x^{-n} = \frac{1}{x^n}$ or $\frac{1}{x^{-n}} = x^n$

Power of power: $(x^m)^n = x^{m*n}$

Power of a quotient: $\left(\frac{x^m}{x^n}\right)^p = \frac{x^{mp}}{x^{np}}$

Power of a product: $(x^m y)^n = x^{mn}y^n$

Zero exponents: $x^0 = 1, x \neq 0$

Exponent Form: $x^{\frac{2}{3}}$ Radical Form: $\sqrt[3]{x^2} or (\sqrt[3]{x})^2$

1. Simplify $(16x^5y^{-3}z^2)^{-1/4}$

$16^{-1/4} x^{-5/4} y^{3/4} z^{-1/2} \rightarrow \frac{y^{3/4}}{2x^{5/4} z^{1/2}}$

2. Simplify $(4x^{-3}y^4z^{-2})^{-3/2}$

$4^{-3/2} x^{9/2} y^{-6} z^3 \rightarrow \frac{x^{9/2} z^3}{8y^6}$

3. Simplify $(8w^7x^{-5}y^3z^{-9})^{-2/3}$

$8^{-2/3} w^{-14/3} x^{10/3} y^{-2} z^6 \rightarrow \frac{x^{10/3} z^6}{4w^{14/3} y^2}$

4. Which expression is equivalent to $\left(\frac{16x^{\frac{1}{6}}y^{-2}}{x^{-\frac{1}{6}}y^6}\right)^{\frac{3}{2}}$?

A. $24x^{\frac{9}{2}}y^{\frac{9}{2}}$

B. $\frac{24x^{\frac{3}{4}}}{y^9}$

C $\frac{64}{x^{\frac{1}{2}}y^8}$

D. $\frac{64x^{\frac{1}{2}}}{y^{12}}$

$\left(\frac{16x^{1/3}}{y^8}\right)^{3/2} =$

$\frac{16^{3/2}x^{1/2}}{y^{12}} =$

$\frac{64x^{1/2}}{y^{12}}$

5. Which expression is equivalent to $(4x)^{1/2} * 36^{1/2}$

A. 12x B. 36x C. $12\sqrt{x}$ D. $24\sqrt{x}$

$4^{1/2}x^{1/2} \cdot 36^{1/2}$

$2x^{1/2} \cdot 6 = 12x^{1/2}$

6. Which expression is equivalent to $\left(\frac{16x^4y^{-2}}{25x^{\frac{1}{2}}y^{-4}}\right)^{-\frac{1}{2}}$?

A. $\frac{5}{4x^{\frac{7}{4}}y}$

B. $\frac{5x^{\frac{7}{4}}}{4y^2}$

C $\frac{12.5}{8x^{\frac{7}{4}}y}$

D. $\frac{16y}{25x^2}$

$\left(\frac{16x^{\frac{7}{2}}y^2}{25}\right)^{-1/2}$

$\frac{16^{-1/2}x^{-\frac{7}{4}}y^{-1}}{25^{-1/2}} = \frac{5}{4x^{7/4}y}$

7. Simplify $\frac{\sqrt[5]{b^3}}{b^{\frac{4}{3}}}$

$\frac{b^{\frac{3}{5}}}{b^{4/3}} = b^{3/5 - \frac{4}{3}} = b^{\frac{9}{15} - \frac{20}{15}}$

$= b^{-\frac{11}{15}} = \frac{1}{b^{11/15}}$

Exponential Functions:

Exponential Growth:
$y = ab^x$ *where* $a > 0$ *and* $0 < b < 1$
b=1+r

Exponential Decay:
$y = ab^x$ *where* $a > 0$ *and* $b > 1$
b=1-r

Compound Interest
$A = P(1+\frac{r}{n})^{nt}$

Interest Compounded Continuously
$A = Pe^{rt}$

Half Life
$y = a\left(\frac{1}{2}\right)^x$

Solving Exponential Equations
$b^x = b^y$ *then* $x = y$ because bases are same
Ex: Solve for x. $10^{3x-1} = 100{,}000$

$10^{3x-1} = 10^5$
3x-1=5
x=2

When bases aren't the same: Isolate the exponential expression, take the log of both sides and solve. Check solutions!!
Ex: $5(10)^{2x} = 60$
Step 1: Isolate the exponential expression.

$(10)^{2x} = 12$

Step 2: Take logarithm of both sides. Remember the exponent gets moved to multiply by the log(base).

$2x * \log(10) = \log(12)$

Step 3: Simplify & Solve.

$\frac{2x*\log(10)}{\log(10)} = \frac{\log(12)}{\log(10)}$
$2x = 1.0792$
x=0.5396

1. In 1950, a U.S. population model was $y = 151(1.013)^{t-1950}$ million people, where t is the year. What did the model predict the U.S. population would be in the year 2000?

$y = 151(1.013)^{2000-1950}$ = 288 million

2. Copper production increased at a rate of about 4.9% per year between 1988 and 1993. In 1993, copper production was approximately 1.801 billion kilograms. If this trend continued, which equation best models the copper production (P) in billions of kilograms, since 1993? (Let t=0 for 1993)

A. $P = 1.801(4.900)^t$
C. $P = 1.801(1.049)^t$
B. $P = 1.801(1.490)^t$
D. $P = 1.801(0.049)^t$

$1 + r = 1 + 0.049$

3. The population of a small town in North Carolina is 4,000, and it has a growth rate of 3% per year. Write an expression which can be used to calculate the town's population x years from now?

$y = 4000(1.03)^x$

4. Alan has just started a job that pays a salary of $21,500. At the end of each year of work, he will get a 5% salary increase. What will his salary be after getting his fifth increase?

$y = 21500(1.05)^x$
$x = 5 \rightarrow$ $27440.05

5. The value, V of a car can be modeled by the function $V(t) = 13000(0.82)^t$ where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation? Annual depreciation: 18%

Monthly $= \frac{18\%}{12} = 1.5\%$

6. The function $V(t) = 1000(1.06)^{2t}$ models the value of an investment after t years.

i. What is the initial value of the investment? 1000

ii. As a percent, what interest rate is the investment earning each year? 12%

Do not worry about this. We did not focus on compound interest

7. If the equation $y = 2^x$ is graphed, which of the following values of x would produce a point closest to the x-axis?

A. ¼ B. ¾ C. 5/3 D. 8/3

Smallest x

8. Suppose a hospital patient receives medication that is used up in the body according to the equation $M = 200(0.8^t)$ with M in milligrams and t in hours. What does the 0.8 represent in the equation?

A. The medication is used up in 0.8 hours.

B. The medication is used up in 0.8 milligrams per hour.

C. The patient started out with 0.8 milligrams of medication.

D. There is 80% of the medication remaining after each hour.

9. Solve $100^{x+6} = 1000^{2x+3}$

$(10^2)^{x+6} = (10^3)^{2x+3}$

$10^{2x+12} = 10^{6x+9}$

$2x + 12 = 6x + 9$

$3 = 4x$

$x = \frac{3}{4}$

10. A city's population, P (in thousands), can be modeled by the equation $P = 130(1.03)^x$ where x is the number of years after January 1, 2000. For what value of x does the model predict that the population of the city will be ***approximately*** 170,000 people?

$170 = 130(1.03)^x$

$\frac{170}{130} = (1.03)^x$

$\log \frac{170}{130} = x \log 1.03$

$x = \frac{\log\left(\frac{170}{130}\right)}{\log 1.03} = 9.08 \text{ yrs}$

11. A new automobile is purchased for \$20,000. If $V = 20{,}000(0.8)^x$ gives the car's value after x years, about how long will it take for the car to be worth half its purchase price?

$10000 = 20000(0.8)^x$

$\frac{1}{2} = (0.8)^x$

$\log \frac{1}{2} = x \log 0.8$

$x = \frac{\log \frac{1}{2}}{\log 0.8} = 3.11 \text{ yrs}$

12. Solve for x: $3^{5x} = 9^{2x-1}$

$3^{5x} = (3^2)^{2x-1}$

$3^{5x} = 3^{4x-2}$

$5x = 4x - 2$

$x = -2$

Solving Advanced Equations:

Direct Variation $y = kx$

"y varies directly with x" Solve: $\frac{y}{x} = \frac{y}{x}$

Ex: y varies directly with x. Find y If y is 2 when x is 3 find y when x is 6.

$\frac{2}{3} = \frac{y}{6}$ y=4

Inverse Variation $y = \frac{k}{x}$

"y varies inversely with x" Solve: $xy = xy$

Ex: Suppose y varies inversely with x. Find x when y is 7, if y is 14 when x is 2.

$x(7) = 2(14)$ x=4

Direct/Inverse Variation (combined) $y = \frac{kx}{z}$

"y varies directly with x and inversely with z"

Ex: If y varies directly as x and inversely as z, and y=24 when x=48 and z=4, find x when y=44 and z=6.

$$\mathbf{24 = \frac{k(48)}{(4)} \rightarrow k = 2 \rightarrow 44 = \frac{2x}{6}}$$

$$\mathbf{x = 132}$$

Solving Rational and Radical Equations.

Ex: Solve $2x = \sqrt{5x-1} + 1$

Step 1: Subtract 1 from each side to isolate the radical term.

$$2x - 1 = \sqrt{5x-1}$$

Step 2: Square both sides to eliminate the radical.

$$4x^2 - 4x + 1 = \sqrt{5x-1}$$

Step 3: Set the right side equal to 0.

$$4x^2 - 9x + 2 = 0$$

Step 4: Solve for x (quadratic so use factoring, graphing or quadratic formula)

$$x = \frac{1}{4} \; and \; x = 2$$

Step 5: Check solutions in the original equation and check for extraneous solutions.

$2\left(\frac{1}{4}\right) = \sqrt{5\left(\frac{1}{4}\right) - 1} + 1$ $2(2) = \sqrt{5(2) - 1} + 1$

$\frac{1}{2} \neq 1\frac{1}{2}$ 4=4

so x=1/4 is **not** a solution. So x=2 is a solution.

The solution ¼ is an extraneous solution because it is a solution to the transformed equation, not to the original equation.

Ex. Solve $\frac{x}{x-1} - 1 = \frac{x}{2}$

Step 1: Get a common denominator, in this case 2(x-1) It will eliminate the denominators altogether.

$$2x - 2(x-1) = x(x-1)$$

Step 2: Simplify.

$$2x - 2x + 2 = x^2 - x$$
$$0 = x^2 - x - 2$$

Step 3: Solve for x.

$$0 = (x-2)(x+1)$$
$$x = 2 \; and \; x = -1$$

Step 4: Check solutions in the original equation and check for extraneous solutions (or excluded values).

$\frac{2}{2-1} - 1 = \frac{2}{2}$ $\frac{-1}{(-1)-1} - 1 = \frac{-1}{2}$

$1{=}1 \; so \; x = 2 \; is \; a \; solution$ $-\frac{1}{2} \neq -1$

$so \; x = -1 \; is \; not \; a \; solution$

The solution -1 is an extraneous solution because -1 is an excluded value.

1. Solve for x: $\frac{x+1}{5} - 2 = \frac{-4}{x}$

LCD: 5x
EV: x=0

$x(x+1) - 10x = -20$
$x^2 + x - 10x + 20 = 0$
$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$
$x=5$ $x=4$ both work

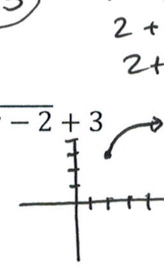
2. Solve for x: $2 + \sqrt{3x+7} = 6$

$\sqrt{3x+7} = 4$
$3x+7 = 16$
$3x = 9$
$x = 3$

$2 + \sqrt{3(3)+7} = 6$
$2 + \sqrt{16} = 6$
$2 + 4 = 6$ ✓

3. For the function $y = \sqrt{x-2} + 3$

i. Sketch a graph

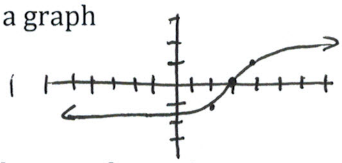


ii. State the transformations

$y = \sqrt{x}$ shifted right 2 and up 3

4. For the function $y = \sqrt[3]{x-3}$

i. Sketch a graph



ii. State the transformations

$y = \sqrt[3]{x}$ shift right 3

5. Solve for x: $\frac{4}{x-2} = \frac{-1}{x-3}$ EV: $x = 2, 3$

$4(x-3) = -1(x-2)$

$4x - 12 = -x + 2$

$5x = 14$

$x = \frac{14}{5}$

6. Solve for x: $\frac{2}{x+2} - \frac{1}{x} = \frac{-4}{x(x+2)}$ LCD $= x(x+2)$

EV: $x = 0, -2$

$2(x) - 1(x+2) = -4$

$2x - x - 2 = -4$

$x - 2 = -4$

$x = -2 \rightarrow$ extraneous

no solution

7. Suppose that y varies inversely with the square of x, and y=50 when x=4. Find y when x=5.

$y = \frac{k}{x^2}$

$50 = \frac{k}{4^2}$

$50 = \frac{k}{16}$

$k = 800$

$y = \frac{800}{x^2}$

$x = 5 \rightarrow y = \frac{800}{25} = 32$

8. Suppose that y varies directly with x and inversely with z^2, and x=48 when y=8 and z=3. Find x when y=12 and z=2.

$y = \frac{kx}{z^2}$

$8 = \frac{48k}{3^2}$

$72 = 48k$

$k = 1.5$

$y = \frac{1.5x}{z^2}$

$12 = \frac{1.5x}{2^2}$

$48 = 1.5x$

$x = 32$

9. A salesperson's commission varies directly with sales. For \$1000 in sales, the commission is \$85.

i. What is the constant of variation (k)?

$C = kS$ $85 = 1000k$

$k = 85/1000 = 0.085$

ii. What is the variation equation?

$C = 0.085S$

iii. What is the commission for a \$2300 sale?

$C = 0.085(2300)$

$= \$195.50$

10. If y varies directly with x and y is 18 when x is 6, which of the following represents this situation?

A.	y=24x	B.	y=3x
C.	y=12x	D.	y=1/3x

$y = kx$

$18 = 6k$

$k = 3$

11. The number of bags of grass seed n needed to reseed a yard varies directly with the area a to be seeded and inversely with the weight w of a bag of seed. If it takes two 3-lb bags to seed an area of 3600 square feet, how many 3-lb bags will seed 9000 square feet?

A.	3 bags	B.	4 bags
C.	5 bags	D.	6 bags

$n = \frac{ka}{w}$

$2 = \frac{3600k}{3}$

$6 = 3600k$

$k = \frac{1}{600}$

$n = \frac{\frac{1}{600}(9000)}{3} = \frac{15}{3} = 5$

12. The volume, V, of a certain gas varies inversely with the amount of pressure, P, placed on it. The volume of this gas is 175 cm³ when 3.2 kg/cm² of pressure is placed on it. What amount of pressure must be placed on 400 cm³ of this gas?

A.	1.31	B.	1.40
C.	2.86	D.	7.31

$V = \frac{k}{P}$

$175 = \frac{k}{3.2}$ $k = 560$

$V = \frac{560}{P}$

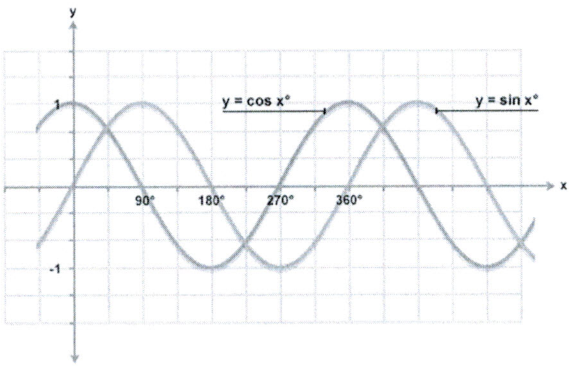
$400 = \frac{560}{P}$

$P = \frac{560}{400}$

$P = 1.4$

Trigonometry:

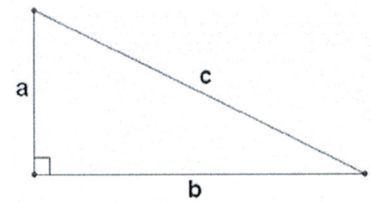
Graphing Sine and Cosine



Amplitude: Distance the max or min is from the midline. Always positive.

Midline: The line that cuts through the middle of the curve, the vertical shift in the curve

Pythagorean Theorem



$a^2 + b^2 = c^2$

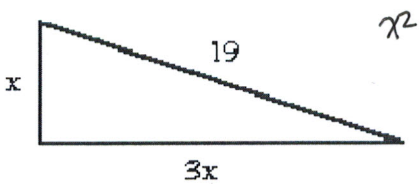
Right Triangle Trig

$$\sin\alpha = \frac{opposite}{hypotenuse}, \cos\alpha \frac{adjacent}{hypotenuse}, and \tan\alpha = \frac{opposite}{adjacent}$$

1. Label the sides of the triangle based on the given angle
2. Set up the trig ratio based on the information given.
3. Solve for the missing side or angle. If solving for a missing side use cross multiplication. If solving for a missing angle, use inverse trig functions.

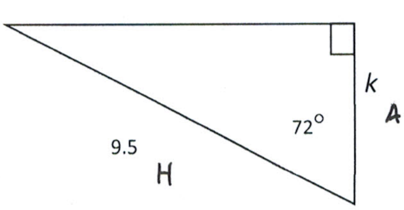
Area of Oblique Triangles Area=(1/2)a* b*sin(C)

1. Find the length of both of the missing sides on the following right triangle:



$x^2 + (3x)^2 = 19^2$
$x^2 + 9x^2 = 361$
$10x^2 = 361$
$x^2 = 36.1$
$x = 6.008$
$3x = 18.025$

2. Find the value of *k*, correct to 1 decimal place. Show all work.

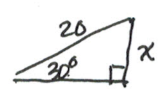


$\cos(72) = \frac{K}{9.5}$

$K = 9.5\cos(72)$

$K = 2.9$

3. An escalator at an airport slopes at an angle of 30° and is 20 m long. Through what height would a person be lifted by travelling on the escalator?



$\sin 30° = \frac{x}{20}$

$x = 20\sin 30 = 10m$

4. The top of a flagpole is connected to the ground by a cable 12 meters long. The angle that the cable makes with the ground is 40°. Find the height of the flagpole.

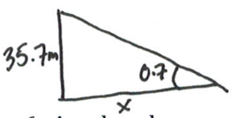
$\sin 40 = \frac{x}{12}$

$x = 12\sin 40$

$x = 7.71\ m$

5. A ship's navigator observes a lighthouse on a cliff. She knows from a chart that the top of the lighthouse

is 35.7 meters above sea level. She measures the angle of elevation of the top of the lighthouse to be 0.7°.The coast is very dangerous in this area and ships have been advised to keep at least 4 km from this cliff to be safe. Is the ship safe?

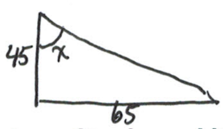


$\tan(0.7) = \frac{35.7}{x}$

$x = \frac{35.7}{\tan(0.7)} = 2921.9\ m$

No, not safe.

6. A school soccer field measures 45 m by 65 m. To get home more quickly, Urooj decides to walk along the diagonal of the field. What is the angle of Urooj's path, with respect to the 45-m side, to the nearest degree?

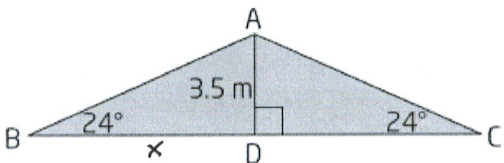


$\tan x = \frac{65}{45}$

$x = \tan^{-1}\left(\frac{65}{45}\right)$

$x = 55.3°$

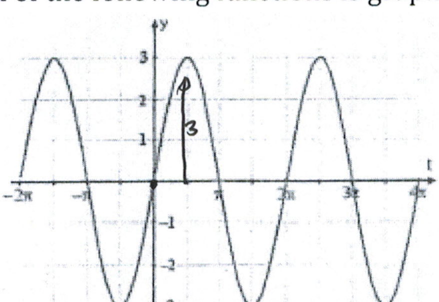
7. A roof is shaped like an isosceles triangle. The slope of the roof makes an angle of 24° with the horizontal, and has an altitude of 3.5 m. Determine the width of the roof, to the nearest tenth of a meter.



$\tan 24 = \frac{35}{x}$

$x = \frac{35}{\tan 24} = 7.86$ $\qquad 2x = 15.7\ m$

8. Which of the following functions is graphed below?

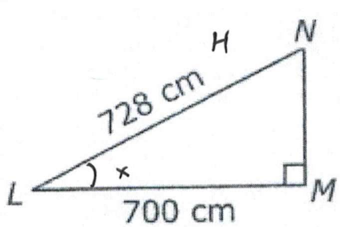


a = 3 because amplitude is 3

sin x because at x=0 function at midline

A. 3sin(x) B. 3cos(x) C. sin(3x) D. cos(3x)

9. In the right triangle LMN, LN=728 cm and LM=700 cm. What is the approximate measure of <NLM?



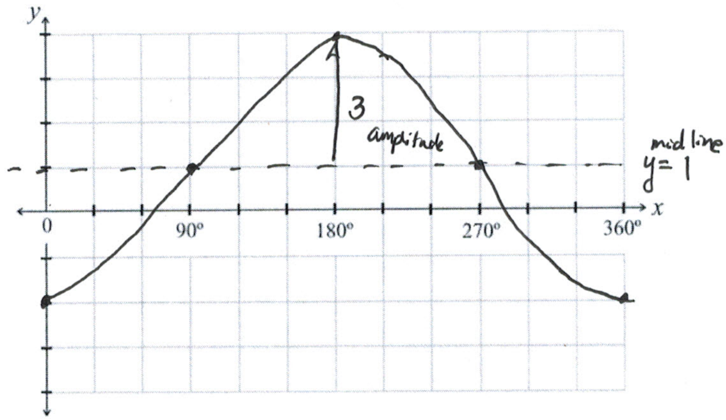
10. What is the a

A

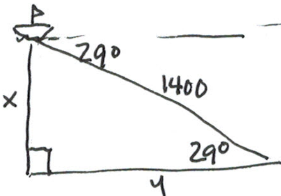
$\cos x = \frac{700}{728}$

$x = \cos^{-1}\left(\frac{700}{728}\right) = 15.9°$

11. Graph $y = -3cos\theta + 1$. Identify they amplitude and midline.



12. Electronic instruments on a treasure-hunting ship detect a large object on the sea floor. The angle of depression is 29°, and the instruments indicate that the direct-line distance between the ship and the object is about 1400 ft. About how far below the surface of the water is the object, and how far must the ship travel to be directly over it?



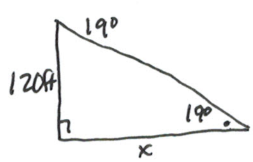
$\sin 29° = \frac{x}{1400}$

$x = 1400 \sin 29° = 678.3$

$\cos 29° = \frac{y}{1400}$

$y = 1400 \cos 29° = 1224.5$

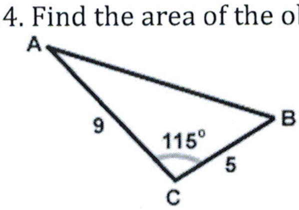
13. From the top of a 120 foot tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?



$\tan 19° = \frac{120}{x}$

$x = \frac{120}{\tan 19} = 348.5\ ft$

14. Find the area of the oblique triangle.



$A = \frac{1}{2}(5)(9) \sin 115$

$= 20.39$