Unit 1: Probability Spring 2016 Name: Key

1. A traveler can choose from four airlines, eight hotels, and three rental car companies. In how many ways can the traveler select an airline, a hotel, and a rental car for his trip?
Fundamental Counting Principle: $4 \times 8 \times 3 = (96)$
2. Ronaldo flips a coin 56 times. He flipped HEADS 22 times. What is the experimental probability that Ronaldo got TAILS? Exp. Prob $= \frac{\#\text{Observances}}{\#\text{Trials}} = \frac{56-22}{56} = \frac{34}{56} = \left(\frac{17}{28} = .607 = 60.7\%\right)$
3. It is equally probable that a pointer on a spinner will land on any one of eight regions, numbered 1 through 8. Find the probability that the pointer will stop on an even number or a number less than 3.
Mut. Exc. Inc.: $P(A \text{ or } B) = P(A) + P(B) - P(A+B) \rightarrow P(\text{Even}) + P(<3) - P(\text{Even}+<3) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}$
4. A spinner is used for which it is equally probable that the pointer will land on any one of six regions. Three of the regions are colored red, two are colored green, and one is colored yellow. If the pointer is spun twice, find the probability it will land on green and then yellow.
Ind. Prob: $P(G) \times P(Y) = \frac{2}{6} \times \frac{1}{6} = \frac{2}{36} = (\frac{1}{18})$
5. What is the probability that a card drawn at random from a standard 52-card deck is either a face card or a red card? $P(\text{Face}) + P(\text{Red}) - P(\text{Face}+\text{Red})$
$\frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52} = (\frac{8}{13})$
6. A bag contains 4 red chips, 3 green chips, 2 blue chips, and 1 black chip. If you select two chips without replacement, what is the probability of getting a green chip and then a black chip?
adjust prob. $P(\text{G first}) \times P(\text{Black Second}) = \frac{3}{10} \times \frac{1}{9} = \frac{3}{90} = (\frac{1}{30})$
7. A bag contains 4 red chips, 3 green chips, 2 blue chips, and 1 black chip. If you select two chips without replacement, what is the probability of getting two black chips?
$P(\text{Bk first}) \times P(\text{Bk second}) = \frac{1}{10} \times \frac{0}{9} = (0)$
8. How many 3-letter codes can be formed using the letters A, B, C, D, and E? No letter can be used more than once. Order matters → Permutation
${}_5P_3 = (60)$
9. In a student government election, 11 members are running for election. Students elect four senators from the group of members. In how many ways can this be done? Not 4 unique titles, Combination.
${}_{11}C_4 = (330)$
10. A club elects a president, vice-president, and secretary. How many ways can the officers be selected if there are 13 members and any member can be elected to each position? No person can hold more than one office. 3 unique titles, Permutation
${}_{13}P_3 = 1716$
11. There are 110 students in the 10th grade. Five of these students will be selected randomly to represent your class on a 5-person bowling team. What is the probability that the team chosen will be you and your 4 best friends? Place on bowling team, not unique titles, Combination
${}_{110}C_5 = 122391522$. Prob $= \frac{1}{{}_{110}C_5} = 8.17 \times 10^{-9}$
12. A hamburger shop sells hamburgers with cheese, relish, lettuce, tomato, onion, mustard, or ketchup. How many different hamburgers can be concocted using any 5 of the extras? order of extras does not matter, Combination
${}_7C_5 = 21$
13. A church has 10 bells in its bell tower. Before each church service 3 bells are rung in sequence. No bell is rung more than once. How many sequences are there? Order Matters. Permutation
${}_{10}P_3 = 720$
14. At Kennedy Middle School, the probability that a student takes Technology and Spanish is 0.087. The probability that a student takes Technology is 0.68. What is the probability that a student takes Spanish given that the student is taking Technology?
Cond. formula $P(A|B) = \frac{P(A+B)}{P(B)}$ $P(\text{Span}|\text{Tech}) = \frac{P(\text{Span}+\text{Tech})}{P(\text{Tech})} = \frac{0.087}{0.68} = (.128)$

15. What is the probability of choosing an ace from a standard deck of cards given that the card you draw is a black card? $P(\text{Ace} \mid \text{Black}) = \frac{P(\text{Ace} + \text{Black})}{P(\text{Black})} = \frac{2/52}{26/52} = \frac{2}{26} = \frac{1}{13}$

16. Seniors at Central High School were surveyed about their grade point averages and about their intentions of higher education after high school graduation. The table below shows results from the survey.

	Attending 4-year college	Not Attending 4-year college	
GPA ≥ 3.0	295	50	345
GPA < 3.0	85	170	255
	380	220	600

17. Find P(student GPA ≥ 3.0).

$\frac{345}{600} = \frac{69}{120} = \frac{23}{40}$

18. Find P(student is attending 4-year college | student GPA < 3.0).

$\frac{P(4\text{yr} + <3.0)}{P(<3.0)} = \frac{85/600}{255/600} = \frac{85}{255} = \frac{1}{3}$

Unit 2: Transformations

Not provided on test

Translation:

$T_{a,b}(x, y) = (x + a, y + b)$

Rotations:

$R_{90°}(x, y) = (-y, x)$

$R_{180°}(x, y) = (-x, -y)$

$R_{270°}(x, y) = (y, -x)$

$R_{-90°}(x, y) = (y, -x)$

Line Reflections:

$r_{x-axis}(x, y) = (x, -y)$

$r_{y-axis}(x, y) = (-x, y)$

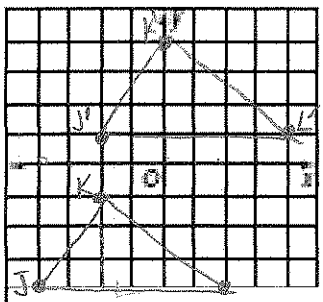
$r_{y=x}(x, y) = (y, x)$

$r_{y=-x}(x, y) = (-y, -x)$

1. Translate the figure below

$\triangle JKL$ with vertices $J(-4, -4)$, $K(-2, -1)$, and $L(2, -4)$ under the translation $(x, y) \rightarrow (x + 2, y + 5)$

J' (-2, 1) K' (0, 4) L' (4, 1)



$J' = (-4 + 2, -4 + 5) = (-2, 1)$

$K' = (-2 + 2, -1 + 5) = (0, 4)$

$L' = (2 + 2, -4 + 5) = (4, 1)$

2. What is the image of $X(3, 5)$ under the translation $(x, y) \rightarrow (x - 4, y + 6)$?

$X' = (3 - 4, 5 + 6) = (-1, 11)$

3. Which of the following will result in a translation?

A. reflecting in two parallel lines

B. reflecting in two intersecting lines

C. reflecting in two perpendicular lines

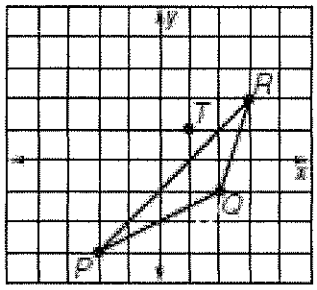
D. turning the figure upside down

4. What is the image of $Y(-4, 7)$ under the translation $(x, y) \rightarrow (x + 3, y - 5)$?

$Y' = (-4+3, 7-5) = (-1, 2)$

5. Rotate the figure 90 degrees according to the directions. List the coordinates of the image.

$\triangle PQR$ with vertices $P(-2, -3)$, $Q(2, -1)$, and $R(3, 2)$ clockwise about the point $T(1, 1)$



$\rightarrow -90$

$R_{0,-90} = (x, y) \rightarrow (y, -x)$

P' (-3, 2)

Q' (-1, -2)

R' (2, -3)

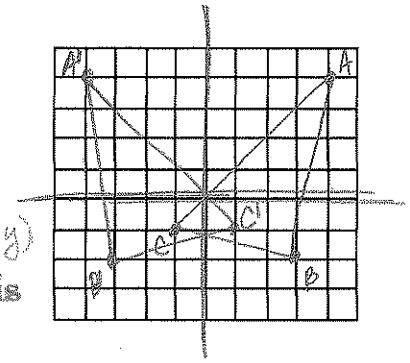
6. Name the image of $C(6, -4)$ under rotation 90° counterclockwise about the origin. $R_{0,90} = (x, y) \rightarrow (-y, x)$ $C'(4, 6)$

7. Name the image of the point (-2, 3) rotated 270 degrees counterclockwise about the origin.

$R_{0,270} = R_{0,90} \rightarrow (x, y) \rightarrow (y, -x) \rightarrow (3, 2)$

8. Graph $\triangle ABC$ with vertices $A(4, 4)$, $B(3, -2)$, and $C(-1, -1)$. Then graph the image of $\triangle ABC$ reflected in the y-axis.

$r_{y\text{-axis}} = (x, y) \rightarrow (-x, y)$ $A'(-4, 4)$ $B'(-3, -2)$ $C'(1, -1$



9. Given $A(3, -7)$, under which reflection is $A'(3, 7)$? $(x, y) \rightarrow (x, -y)$

(A.) reflection in the x-axis

B. reflection in the y-axis

C. reflection in the origin

D. reflection in $y = x$

10. Name the coordinates of the image of $Q(6, -4)$ reflected in the x-axis. $r_{x\text{-axis}}(x, y) \rightarrow (x, -y)$ $Q'(6, 4)$

11. Find the coordinates of the image and the measure of the angle of rotation if $\triangle RST$ with $R(5, 3)$, $S(7, 8)$, and $T(10, 1)$ is reflected in the line $y = x$ and then in the x-axis.

$r_{y=x}(x, y) \rightarrow (y, x)$

$r_{x\text{-axis}}(x, y) \rightarrow (x, -y)$

Composition $(x, y) \rightarrow (y, -x) = R_{0,90}$ ↺ (CCW)

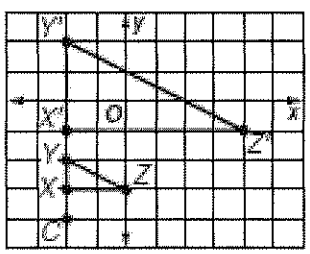
✱ 12. Find the scale factor if $\triangle X'Y'Z'$ is the image of $\triangle XYZ$ under a dilation with center C.

A. 6

B. 3

C. 2

D. $\frac{1}{3}$



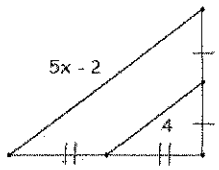
We always dilate with center at origin

13. Find the image of $A(-2, 3)$ reflected in the line $y = -x$ and then in the x-axis. $r_{y=-x}\ (x, y) \longrightarrow (-y, -x)$
$r_{x\text{-axis}}\ (x, y) \longrightarrow (x, -y)$ $\Rightarrow (x, y) \mapsto (-y, x)$
$A'(-3, 2)$

14. $\triangle XYZ$ has vertices $X(-1, 3)$, $Y(5, 7)$, and $Z(2, -4)$. Find the coordinates of the image of $\triangle XYZ$ after a dilation centered at the origin with a scale factor of 4.
$(x, y) \longrightarrow (4x, 4y)$
$X'(-4, 12)$
$Y'(20, 28)$
$Z'(8, -16)$

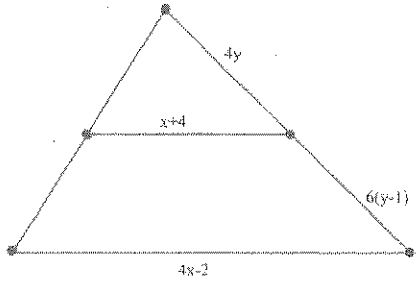
Unit 3: Triangles and Congruence

1. Solve for x in the figure below



$5x - 2 = 2(4)$
$5x - 2 = 8$
$5x = 10$
$x = 2$

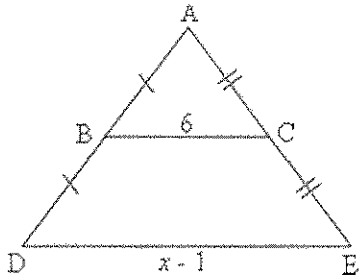
2. Solve for x and y in the triangle if the segment in the middle is a midsegment.



$2(x+4) = 4x - 2$
$2x + 8 = 4x - 2$
$10 = 2x$
$x = 5$

$4y = 6(y-1)$
$4y = 6y - 6$
$6 = 2y$
$y = 3$

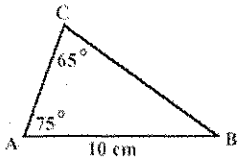
3. Solve for x if BC is a midsegment.



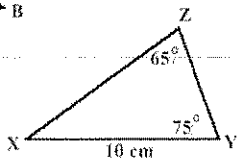
$2(6) = x - 1$
$12 = x - 1$
$x = 13$

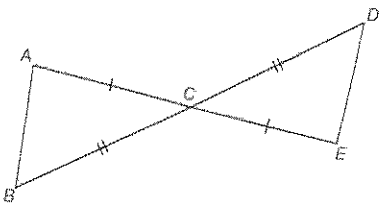
Determine if the triangles are congruent. If yes, make a congruency statement and give the reason why they are congruent. If they are not congruent, write "not congruent".

4 congruence theorems
SSS
SAS
AAS
ASA

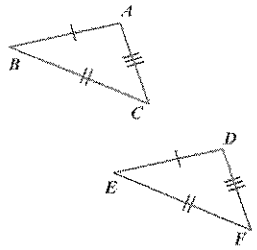
4. 

$\triangle ABC \cong \triangle$ YXZ by AAS



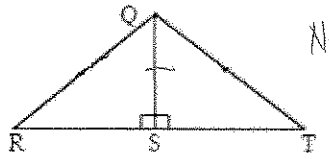
5. 

$\Delta ABC \cong \Delta$ EDC by SAS

6. 

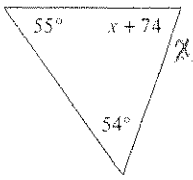
$\Delta ABC \cong \Delta$ DEF by SSS

7. What additional information do you need to prove that triangle QRS is congurent to triangle QTS by AAS?



Need $\angle R \cong \angle T$

8. Solve for x



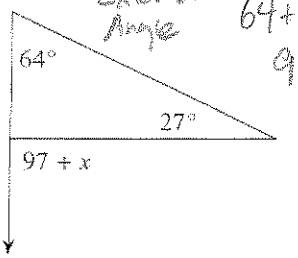
Triangle Sum

$x+74+54+55=180$

$x+183=180$

$x=-3$

9. Solve for x.



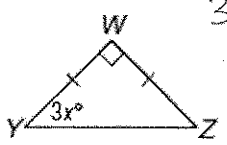
Exterior Angle

$64+27=97+x$

$91=97+x$

$x=-6$

10. Solve for x in the isosceles triangle.

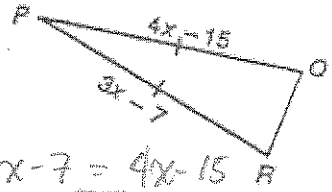


$3x+3x+90=180$

$6x=90$

$x=15$

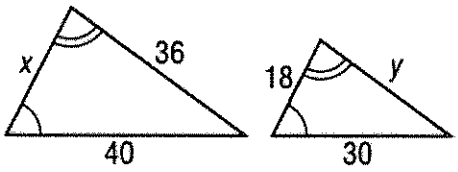
11. Solve for x in the isosceles triangle.



$3x-7=4x-15$

$8=x$

12. Solve for x and y in the following similar figures.



$\frac{30}{40}=\frac{18}{x}$

$30x=720$

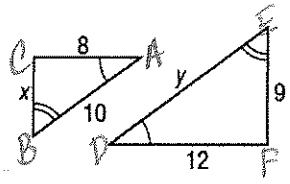
$x=24$

$\frac{30}{40}=\frac{y}{36}$

$10.80=40y$

$y=27$

13. Solve for x and y in the following similar figures and then find the perimeter of each triangle.



$\Delta ABC \sim \Delta DEF$

$\frac{AC}{DF}=\frac{BC}{EF}$

$\frac{8}{12}=\frac{x}{9}$

$12x=72$

$x=6$

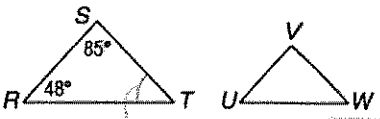
$\frac{AC}{DF}=\frac{AB}{DE}$

$\frac{8}{12}=\frac{10}{y}$

$8y=120$

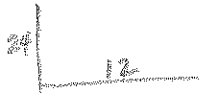
$y=15$

14. If $\triangle RST \sim \triangle UVW$, find $m\angle W$.



$48+85+m\angle T=180$ $133+m\angle T=180$ $(m\angle T=47)$ $\angle T \cong \angle W$ $(m\angle W=47)$

15. A tree 24 feet tall casts a shadow 12 feet long. Brad is 6 feet tall. How long is Brad's shadow? (draw a diagram and solve)



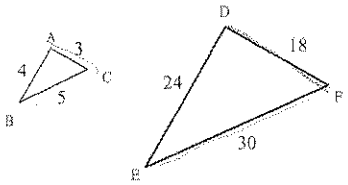
$\frac{24}{6}=\frac{12}{x}$ $24x=72$ $(x=3)$

16. Solve the proportion $\frac{x-2}{4}=\frac{x+4}{2}$ $2(x-2)=4(x+4)$ $2x-4=4x+16$ $2x=-20$ $(x=-10)$

Determine if the triangles are <u>similar</u>. If yes, make a similarity statement and give the reason why they are similar. If they are not congruent, write "not similar".

3 Similarity Theorems
SSS
SAS
AA

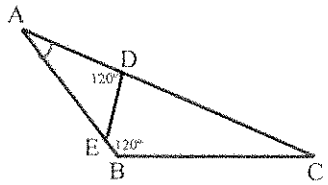
17.



$\triangle DFE \sim \triangle$ ACB by SSS

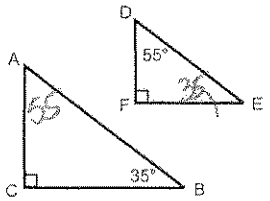
$\frac{3}{18}=\frac{4}{24}=\frac{5}{30}$ $\rightarrow \frac{1}{6}$ ✓

18.



$\triangle DBC \sim \triangle$ ADE by AA

19.



$\triangle ACB \sim \triangle$ DFE by AA

Unit 4: Quadratics

1. Multiply the following polynomials

a. $(x+1)(2x-7)$ $2x^2-7x+2x-7$ $(2x^2-5x-7)$

b. $(-x-5)(x^2-3x+2)$ $-x^3+3x^2-2x-5x^2+15x-10$ $=-x^3-2x^2+13x-10$

c. $(2x+y)(x-3y)$ $2x^2-6xy+xy-3y^2$ $2x^2-5xy-3y^2$

2. Factor the following polynomials. Don't forget to do GCF first!

a. $3x^2+10x-25$ Slip+Slide ① $x^2+10x-75$ ② $(x+15)(x-5)$ ③ $(x+\frac{15}{3})(x-\frac{5}{3})$ ④ $(x+5)(x-5)$ ⑤ $(x+5)(3x-5)$

b. x^2+4x+5 $b^2-4ac=16-4(1)(5)=-4$ No solutions. PRIME

c. $2x^3-10x^2-50x+250$ Grouping GCF $2(x^3-5x^2-25x+125)$ $2(x^2(x-5)-25(x-5))$ $2(x^2-25)(x-5)$ $2(x+5)(x-5)(x-5)$ $2(x+5)(x-5)^2$

d. $9x^2-36y^2$ GCF Diff. Sq. $9(x^2-4y^2)$ $9(x-2y)(x+2y)$

3. Identify the solutions to the quadratics $x^2 - 11x + 28 = 0$.

$(x-7)(x-4)=0$

$x=7$ $x=4$

4. What is the discriminant of the function $x^2 - 49 = 0$. What does this tell you about the graph?

b^2-4ac

$0^2-4(1)(-49)=196$ → 2 real solutions

5. Find the zeros of the quadratic $x^2 + x = 12$.

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ $x=3$

6. Solve the quadratic $x^2 - 10x + 35 = 7x - 35$.

$x^2-17x+70=0$

$(x-10)(x-7)=0$

$x=10$ $x=7$

7. Solve for p in the quadratic $3 = 16p - 20p^2$

$20p^2-16p+3=0$

$p^2-16p+60$

$(p-10)(p-6)$

$(p-\frac{10}{20})(p-\frac{6}{20})$ → $(p-\frac{1}{2})(p-\frac{3}{10})$

$(2p-1)$ $(10p-3)$

$2p=1$ $10p=3$

$p=\frac{1}{2}$ $p=\frac{3}{10}$

8. Solve the system defined by $\begin{cases} f(x)=x^2-2x+2 \\ g(x)=12-5x \end{cases}$

9. An acorn, falling from the top of a tree is modeled by the equation $h = -16t^2 + 45$. Before it can hit the ground a squirrel hiding on a lower branch jumps out and intercepts it. If the squirrel's movement is modeled by the equation $h = -3t + 32$, at what height did the squirrel intercept the acorn?

$Y_1 = -16t^2+45$

$Y_2 = -3t+32$

intersect at $(1, 29)$ height

10. Graph $y \geq x^2 - 2x - 24$

$(x-6)(x+4)$

solid

use $(0,0)$ $0 \geq -24$ true, shade in side

zeros: $(6,0)$, $(-4,0)$

AOS: $x=\frac{-b}{2a}=\frac{-(-2)}{2(1)}=1$

$x=1$

vertex: $(1,-25)$

y-int $(0,-24)$

0 intercepts: $(5,-9)$ $(-3,-9)$

11. Graph $y < 2x^2 - 3x - 7$

AOS: $\frac{3}{2(2)}=\frac{3}{4}$

Vertex: $(\frac{3}{4}, -8.125)$

y-int $(0,-7)$

$-1.26, 0)$

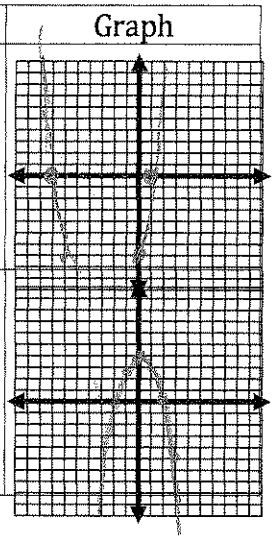
$2.76, 0$

dashed

try $(0,0)$

$0 < -7$ not true shade outside

12. Identify axis of symmetry, vertex, minimum or maximum, zeros and y-intercept of a quadratic function in standard form. Then graph the quadratic.

Equation	A.O.S	Vertex	Zeros	Y-Intercept	Graph
#1 $y = x^2 + 6x - 7$ $9 - 16 = -7$ $(x+7)(x-1)$	$x = \frac{-b}{2} = -3$	$(-3, -16)$	$(-7, 0)$ $(1, 0)$	$(0, -7)$	
#2 $y = -x^2 + 4$	$x = \frac{0}{2} = 0$	$(0, 4)$	$(2, 0)$ $(-2, 0)$	$(0, 4)$	

13. Two rockets are launched above a lake. The equations of the paths are modeled below.

Rocket A:

$H = -16t^2 - 4t + 382$

Rocket B:

$-16t^2 + 64t + 80$

Time	Height
½	108
2	144
3	128
4.5	44

a. Which rocket flies the highest? (A) max: 382.25
B max: 144

b. Which rocket is in the air the longest?

A - positive zero: 4.76

(B) - positive zero: 5.00