**Honors Math 2 – Unit 5 –Exponential Functions** Name: ____________________
Notes and Activities Date: ____________ Pd: _____

Unit Objectives:
**Objectives:**
**N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
**F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
**F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
**A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
**A-CED.1** Create equations and inequalities in one variable and use them to solve problems.
**F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
**A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

| Date | Lesson |
|---|---|
| End of 3Q | Properties of Exponents (Review rules) |
| Mon 4/4 | Rational Exponents |
| Tue 4/5 | Radical→Rational Exponents |
| Wed 4/6 | Solving Equations with Rational Exponents |
| Thur 4/7 | ***Quiz***<br>Common Logs |
| Fri 4/8 | Solving for Exponents |
| Mon 4/11 | Review and Practice |
| Tue 4/12 | Modeling Growth / Decay |
| Wed 4/13 | Modeling Growth / Decay |
| Thu 4/14 | Review |
| Fri 4/15 | **TEST** |

| **Rational Exponents** |
|---|

**Warm-up (**Properties of Exponents…with FRACTIONS!)

Even though they seem more complicated, fractions are numbers too. You can use all the same properties with fraction (rational) exponents as you can with integer exponents. Write down those properties first.

$a^m \cdot a^n =$ ________ $\frac{a^m}{a^n} =$ ________ $\frac{1}{a^n} =$ ________ $(a^m)^n =$ ________ $(a \cdot b)^n =$ ________

**warm-up continues on the next page**

Work with a partner to write each expression in simplified form (one coefficient, each base used only once with one exponent each, no negative exponents). Each of you should complete one column, taking turns. Check each other's work.

| COLUMN A | COLUMN B |
|---|---|
| $\left(x^{\frac{2}{3}}\right)^{-3}$ | $\left(x^{-\frac{4}{7}}\right)^{7}$ |
| $\left(3x^{\frac{2}{3}}\right)^{-1}$ | $5\left(x^{\frac{2}{3}}\right)^{-1}$ |
| $(-27x^{-9})^{\frac{1}{3}}$ | $(-32x^{15})^{\frac{1}{5}}$ |
| $\left(\frac{x^3}{x^{-1}}\right)^{-\frac{1}{4}}$ | $\left(\frac{x^2}{x^{-10}}\right)^{\frac{1}{3}}$ |
| $\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6}$ | $\left(x^{\frac{2}{3}}y^{-\frac{1}{6}}\right)^{-12}$ |
| $\left(\frac{x^{\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{12}$ | $\left(\frac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15}$ |

## Rational Exponents: Practice 1

Simplify each of the following using the properties of exponents.

(a) $27^{\frac{2}{3}}$

(d) $3y^{\frac{1}{4}} \cdot 2y^{\frac{2}{3}}$

(g) $\left(16x^4y^8\right)^{\frac{1}{2}}$

(b) $27^{-\frac{4}{3}}$

(e) $\dfrac{4x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$

(h) $\dfrac{x^5}{x^{\frac{1}{3}}}$

(c) $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

(f) $x^{\frac{1}{3}}\left(2x^{\frac{1}{2}} - x^{\frac{1}{4}}\right)$

(i) $\dfrac{2x^2y^{\frac{1}{3}}}{4^{-\frac{1}{2}}x^{\frac{1}{2}}y^2}$

**For #67-90, simplify the exponential expressions. Your answer should only have positive exponents.**

**67.** $x^{1/4}x^{-5/4}$

**68.** $2^{2/3}2^{-5/3}$

**69.** $\dfrac{p^{5/3}}{p^{2/3}}$

**70.** $\dfrac{q^{5/4}}{q^{1/4}}$

**71.** $(y^{1/5})^{10}$

**72.** $(x^{1/2})^8$

**73.** $6^{-1/5}6^{3/5}$

**74.** $a^{-1/3}a^{2/3}$

**75.** $\dfrac{4t^{-1/3}}{t^{4/3}}$

**76.** $\dfrac{5s^{-1/3}}{s^{5/3}}$

**77.** $(a^{1/3}a^{1/4})^{12}$

**78.** $(x^{2/3}x^{1/2})^6$

**79.** $(5a^2c^{-1/2}d^{1/2})^2$

**80.** $(2x^{-1/3}y^2z^{5/3})^3$

**81.** $\left(\dfrac{x^{-2/3}}{y^{-3/4}}\right)^{12}$

**82.** $\left(\dfrac{m^{-1/4}}{n^{-1/2}}\right)^{-4}$

**83.** $\left(\dfrac{16w^{-2}z}{2wz^{-8}}\right)^{1/3}$

**84.** $\left(\dfrac{50p^{-1}q}{2pq^{-3}}\right)^{1/2}$

**85.** $(25x^2y^4z^6)^{1/2}$

**86.** $(8a^6b^3c^9)^{2/3}$

**87.** $(x^2y^{-1/3})^6(x^{1/2}yz^{2/3})^2$

**88.** $(a^{-1/3}b^{1/2})^4(a^{-1/2}b^{3/5})^{10}$

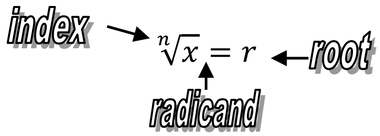
**89.** $\left(\dfrac{x^{3m}y^{2m}}{z^{5m}}\right)^{1/m}$

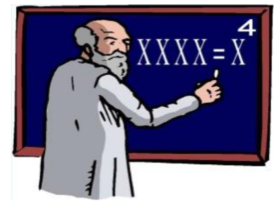
**90.** $\left(\dfrac{a^{4n}b^{3n}}{c^n}\right)^{1/n}$

# Day 3: Radicals to Rational Exponents

The lesson:

We are familiar with taking square roots ($\sqrt{\ }$) or with taking cubed roots ($\sqrt[3]{\ }$), but you may not be as familiar with the elements of a radical.




An index in a radical tells you how many times you have to multiply the root times itself to get the radicand. For example, in the equation $\sqrt{81} = 9$, 81 is the radicand, 9 is the root, and the index is 2 because you have to multiply the root by itself twice to get the radicand ($9 \cdot 9 = 9^2 = 81$). When a radical is written without an index, there is an understood index of 2.

$\sqrt[3]{64} = ?$
Radicand = ______
Index= ______
Simplify:

$\sqrt[5]{32x^5} = ?$
Radicand= ______
Index= ______
Simplify:

**You try: Rewrite each of the following expressions in radical form.**

| | | | |
|---|---|---|---|
| $x^{\frac{3}{2}}$ | $(-27)^{\frac{2}{3}}$ | $(16x)^{\frac{5}{4}}$ | $y^{-9/8}$ |
| $2a^{\frac{1}{4}}$ | $4^{\frac{-7}{2}}$ | $\mathbf{(3^{\frac{2}{5}})^5}$ | $x^{1.2}$ |

**Now, reverse the rule you developed to change radical expressions into rational expressions.**

| | | |
|---|---|---|
| $\sqrt[5]{2}$ | $(\sqrt[3]{6})^5$ | $(\sqrt{5})^7$ |
| $\sqrt{7}$ | $(\sqrt[4]{9^3})$ | $(\sqrt[7]{3x})^2$ |

**Examples: Write with a rational exponent. Simplify.**

| | | |
|---|---|---|
| $\sqrt[5]{243y^5}$ | $\sqrt[4]{1296m^4n^8}$ | $\sqrt{144v^8}$ |

**Practice: Write with a rational exponent. Simplify.**

| | | |
|---|---|---|
| $\sqrt{16x^2}$ | $\sqrt{8x}$ | $\sqrt{15x^3}$ |
| $\sqrt[3]{-8}$ | $\sqrt[3]{80n^5}$ | $\sqrt[4]{96}$ |
| $\sqrt[4]{81}$ | $\sqrt[5]{486}$ | $\sqrt[3]{-40}$ |
| $\sqrt[3]{18x^4}$ | $\sqrt[4]{64x^3}$ | $\sqrt[5]{-32x^3y^6}$ |
| $\sqrt[3]{81x^3y^2z^4}$ | $\sqrt[3]{192x^5y^7z^2}$ | $\sqrt[4]{1875x^4z^2}$ |

Now, write with rational exponents first…then use exponent rules and simplify.

| | | |
|---|---|---|
| $\frac{12\sqrt[3]{y}}{4\sqrt{y}}$ | $\left(\frac{\sqrt[3]{a^2}}{\sqrt{b}}\right)^{-6}$ | $(2\sqrt[4]{a})^3 \cdot \sqrt{a^3}$ |
| $\sqrt[4]{x^{12}} \cdot \sqrt{y^{-2}}$ | $\frac{\sqrt{64x^3}}{\sqrt[3]{512x^9}}$ | $\sqrt[4]{625x^8}$ |
| $\sqrt[7]{x^2} \cdot \sqrt[14]{x^3}$ | $\frac{1}{\sqrt[3]{-27x^9}}$ | $\left(\sqrt{x} \cdot \sqrt[3]{y^2}\right)^{-6}$ |

## Day 3 Classwork

**Rewrite the expression using rational exponent notation.**

**1.** $\sqrt[3]{7}$ **2.** $(\sqrt[3]{6})^2$ **3.** $(\sqrt[5]{14})^4$

**4.** $(\sqrt[7]{-21})^3$ **5.** $(\sqrt[8]{11})^7$ **6.** $(\sqrt[9]{-2})^4$

**Rewrite the expression using radical notation.**

**7.** $17^{1/3}$ **8.** $44^{1/6}$ **9.** $33^{2/3}$

**10.** $9^{5/3}$ **11.** $(-28)^{7/5}$ **12.** $39^{4/7}$

**Evaluate the expression without using a calculator.**

**13.** $(\sqrt[3]{8})^2$ **14.** $(\sqrt[4]{16})^3$ **15.** $(\sqrt[4]{81})^4$

**16.** $36^{3/2}$ **17.** $4^{5/2}$ **18.** $27^{2/3}$

**19.** $125^{4/3}$ **20.** $(-8)^{1/3}$ **21.** $(-32)^{3/5}$

**Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.**

**22.** $\sqrt[3]{38}$ **23.** $\sqrt[6]{112}$ **24.** $\sqrt[7]{-215}$

**25.** $(241)^{1/5}$ **26.** $(-133)^{1/3}$ **27.** $(69)^{1/4}$

**28.** $(96)^{2/3}$ **29.** $(356)^{5/9}$ **30.** $(-2427)^{4/7}$

# Solving Equations with Rational Exponents

You know a lot about inverses in mathematics – we use them every time we solve equations.

Write down the inverse operation for each of the following (there could be more than one correct answer) and then give a definition for "inverse" in your own words.

If you get stuck, it may be helpful for you to write the expression out and think what you would do to solve an equation that had that expression on one side of the equation.

| The phrase… | Is the expression… | And its inverse is… |
|---|---|---|
| adding 5 to a number | $x+5$ | Subtracting 5 from a number |
| subtracting 7 from a number | | |
| multiplying a number by ½ | | |
| Multiplying a number by $\frac{2}{5}$ | | |
| dividing a number by 3 | | |
| squaring a number | | |
| Taking the square root of a number | | |
| Raising a number to the 5th power | | |
| Taking the 5th root of a number | | |
| Raising a number to the $\frac{2}{5}$ power | | |

An "inverse" is…

Make sure you have all of the answers correct to the table above before going on. Now that you're finished, we're going to combine our skills from this unit so far to solve some more challenging equations.

| Skill | Partner A | Partner B |
|---|---|---|
| 1 | $\sqrt[4]{a} = 14$ | $\sqrt[5]{b} = 50$ |
| 2 | $\sqrt[5]{a^9} = 26$ | $\sqrt[4]{b^3} = 27$ |
| 3 | $\left(\sqrt[7]{a}\right)^3 = 21$ | $\left(\sqrt{b}\right)^5 = 12$ |
| 4 | $a^{\frac{1}{5}} = 50$ | $b^{\frac{1}{4}} = 14$ |
| 5 | $a^{\frac{3}{4}} = 27$ | $b^{\frac{9}{5}} = 26$ |
| 6 | $\left(a^{\frac{1}{2}}\right)^5 = 12$ | $\left(b^{\frac{1}{7}}\right)^3 = 21$ |

Work with a partner – Decide who will be Partner A and who will be Partner B.

Compare the answers you got when you practiced Skills 1-3 with the answers you got when you practiced Skills 4-6. Work with your partner to explain your findings.

The previous problems only had one step. You cannot do this step until the radical or rational exponent is isolated on one side of the equation. You can isolate the radical using the inverses discussed at the beginning of the lesson. There are also some problems below in which the rational exponent or radical is applied to the entire side of the equation. Only in these situations will you undo the rational exponents or radicals first. Before solving the entire problem, make sure you know what the first step will be.

$$x^{1/4} - 2 = 3$$

Step 1

$$4x^7 - 6 = -2$$

Step 1

$$3\left(x^{2/3} + 5\right) = 207$$

Step 1

$$1450 = 800\left(1 + \frac{x}{12}\right)^{7.8}$$

Step 1

$$14.2 = 222.1 \cdot x^{3.5}$$

Step 1

$$3x^{3/4} + 5 = 53$$

Step 1

$$x^{1/2} - 5 = 0$$

Step 1

$$(2x + 7)^{1/2} = 3$$

Step 1

$$\sqrt[3]{x - 2} = 4$$

Step 1

$$\sqrt{a + 2} - 2 = 12$$

Step 1

$$\sqrt{2x - 5} = 9$$

Step 1

$$\sqrt[4]{3x + 1} - 5 = 0$$

Step 1

The next few problems are…different. We're going to come across some equations that have no solution and some that have two solutions. Remember, you can always check your answers by substituting your solution into the equation to make sure it works. In fact, you really ***need*** to check your answers to these problems! When we solve an equation correctly, but the answer doesn't work when we check it, we call the solution extraneous.

$\sqrt{a+2}-2=a$

$\sqrt{3x-2}=-5$

$(2x+7)^{1/2}-x=2$

$3x^{4/3}+5=53$

Applications of Equations with Rational Exponents or Radicals.

The distance between two points is $5\sqrt{2}$. If one of the points is located at $(4,2)$ and the other point has a x-value of $-1$, what are the possible y-values of the other point?

The volume of a sphere is 2145. If the formula $V=\frac{4}{3}\pi r^3$ is used to calculate the volume of a sphere, what is the radius of the sphere?

The equation $v=\sqrt{2.5r}$ allows you to calculate the maximum velocity, v, that a car can safely travel around a curve with a radius of $r$ feet. This is used by the Department of Transportation to determine the best speed limit for a given stretch of road. If a road has a speed limit of 45 mph, what is the tightest turn on that road?

## More Practice Solving by Changing the Base

Solve each of the following for **x**:

| | | | | | |
|---|---|---|---|---|---|
| a) | $3^x = 9$ | b) | $2^x = 8$ | c) | $5^x = 25$ |
| d) | $4^x = 64$ | e) | $2^x = \frac{1}{8}$ | f) | $3^x = \frac{1}{27}$ |
| g) | $10^x = \frac{1}{100}$ | h) | $7^x = 1$ | i) | $2^{x+1} = 16$ |
| j) | $3^{x-2} = 9$ | k) | $5^{x+3} = 25$ | l) | $2^{x-7} = 8$ |
| m) | $3^x + 4 = 13$ | n) | $2^x - 6 = 10$ | o) | $5^x + 3 = 28$ |
| p) | $4^{x+2} + 3 = 19$ | q) | $3^{x-2} = \frac{1}{3}$ | r) | $2^{x+7} - 8 = 24$ |
| s) | $3^{x-5} = \frac{1}{9}$ | t) | $5^{2x+1} = 125$ | u) | $\left(\frac{1}{2}\right)^{x+3} = 8$ |

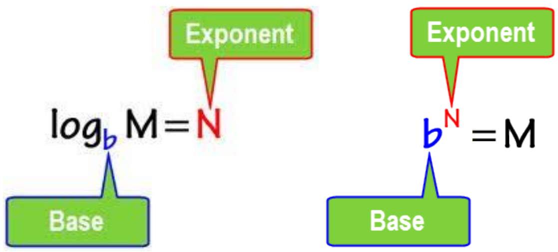
## Common Logs

The inverse of an exponential function is a ________________________________.

A logarithm is defined as follows:

| |
|---|
| The logarithm base *b* of a positive number *y* is defined as follows:<br>If $y = b^x$, then $\log_b y = x$.<br>Where b ≠ 1 and b > 0 |

**Switching between logarithmic form and exponential form**




**Convert exponential form to log form**

1. $x^3 = 17$

2. $\left(\frac{1}{5}\right)^x = \frac{1}{25}$

3. $2^x = 32$

**Convert log form to exponential form**

4. $log_4 x = 12$

5. $log_7 3x = y$

6. $log_u \frac{15}{16} = v$

1. Write the following in exponential form:

(a) $\log_3 x = 9$

(b) $\log_2 8 = x$

(c) $\log_3 27 = x$

(d) $\log_4 x = 3$

(e) $\log_2 y = 5$

(f) $\log_5 y = 2$

2. Write the following in logarithm form:

(a) $y = 3^4$

(b) $27 = 3^x$

(c) $m = 4^2$

(d) $y = 3^5$

(e) $32 = x^5$

(f) $64 = 4^x$

## Solving for Variables in the Exponent

When asked to solve an exponential equation such as $2^{x+6} = 32$ or $5^{2x-3} = 18$, the first thing we need to do is to decide which way is the "best" way to solve the problem.

| Solving Exponential Equations with the Same Base | Solving with Logs |
|---|---|
| **Question: Can you rewrite both sides so that they have the same base?** | |
| **Yes:** Solve by changing the base<br>$2^{x+6} = 32$<br>**Property:** If $B^N = B^M$, then N=M<br><br>**Solve:** | **No:** Solve with Logs<br>$5^{2x-3} = 18$<br>**Property:** $B^N=C^M \rightarrow (N)\log_{10}B=(M)\log_{10}C$<br><br>**Solve:** |

**Example:** Solve $9^{2x-5} = 27$

**Example:** Solve $8^{4x+1} = 205$

**Example:** Solve $5^{3x+7} = 311$

**Example:** Solve $25^{2x-1} = 125^{3x+4}$

**Example:** Solve $8^{2x-3} = \left(\frac{1}{16}\right)^{x-2}$

# Practice Solving for Exponents

1. $4^{3x+5} = 8^{4x-3}$

5. $9^{4x-1} = 27^{5-x}$

2. $5^{6x+1} = 761$

6. $2^{5x-2} = 176$

3. $9^{2x+3} = \left(\frac{1}{27}\right)^{3x+1}$

7. $4^{2x+3} = \left(\frac{1}{32}\right)^{3x+1}$

4. $3^{2x-5} = 87$

8. $7^{4x+3} = 523$

**Independent Practice Solving Exponential Equations with Logs**

1) $16^{b} = 88$

2) $13^{v} = 76$

3) $2 \cdot 20^{n} = 18$

4) $3^{x} - 2 = 11$

5) $6 \cdot 14^{k} + 4 = 76$

6) $6 \cdot 17^{a} + 4 = 20$

7) $2^{x-1} = 23$

8) $5^{x-6} = 18$

9) $3 \cdot 18^{n+8} = 1$

10) $9^{5k} + 8 = 50$

# Practice Solving Radical Equations

1. $\sqrt{x+2}=3$

2. $\sqrt{4x+9}=5$

3. $\sqrt[3]{5-11x}=3$

4. $\sqrt[3]{x^2-1}=2$

5. $\sqrt{x+7}=x-5$

6. $\sqrt{x+5}=x-1$

7. $\sqrt{2x+15}=x+6$

8. $(3x-1)^{3/2}=27$

9. $\sqrt{x+5}=4$

10. $\sqrt[3]{6-3x}=2$

11. $\sqrt{x-7}=-9$

12. $(x+3)^{2/3}=-1$

13. $\sqrt{3x+19}=x-3$

14. $(12-x)^{3/2}=-1$

15. $(5+2x)^{2/3}=9$

16. $\sqrt[3]{12+x}=-3$

17. $\sqrt{10-3x}=\sqrt{x+2}$

18. $(x+2)^{5/2}=-1$

19. $(x-5)^{3/2}=8$

20. $\sqrt[3]{4x-1}=5$

21. $\mathbf{\sqrt{13-x}+x=7}$

22. $\mathbf{\sqrt{x+1}-3=2}$

# Investigation : Exponential Growth & Decay

Materials Needed:
- Graphing Calculator (to serve as a random number generator)

To use the calculator's random integer generator feature:

1. Type any number besides zero into your calculator, press [STO→] , [MATH], [←] , [ENTER], [ENTER]


2. Press [MATH] [←] [5] [1] [,] [6] [)]


You can use numbers other than 1 and 6 here. The calculator will choose numbers between and including these numbers when you press enter. Continue pressing enter for more numbers.

Investigation:
1. Choose a recorder to collect the class's data on the board. You'll copy the data down in your table later.
2. Everyone in the class should stand so that the recorder can count everyone and record the number of people standing in a table for "Stage 0".
3. Use your calculator to find a random integer between 1 and 6. If you roll a 1, sit down. Before proceeding, allow time for the recorder to count the number of people still standing. When the recorder is finished counting, (s)he will let you know.
4. Repeat step 3 until fewer than 3 people are standing (or you run out of room on the table).
5. Record the data in your table.

| Stage | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| People Standing | | | | | | | | | | | |

Questions:
1. What is your initial value for this set of data? What does it represent in the investigation?

2. Would it make more sense to find a common ratio (r) or common difference (d) for this data? Explain.

3. Based on your answer to Question 2, find the r OR d for the data you collected. Show the process you used to do so.

4. Could you estimate your answer to Question 3 without conducting the exploration? If so, how?

5. Write a recursive (NOW-NEXT) function that would help you make predictions for this data.

6. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the stage of the investigation and let f(x) equal the number of people standing in that stage.

Investigation 2:
Half of a radioactive substance decays every 53 years. How much will remain of a 12 milligram sample after 530 years?

Complete the table.

| Years | 0 | 53 | 106 | 159 | 212 | 265 | 318 | 371 | 424 | 477 | 530 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Remaining radioactive substance | | | | | | | | | | | |

Questions:

7. What is your initial value for this set of data? What does it represent in the investigation?

8. Would it make more sense to find a common ratio (r) or common difference (d) for this data? Explain.

9. Based on your answer to Question 8, find the r OR d for the data you collected. Show the process you used to do so.

10. Could you estimate your answer to Question 9 without filling in the table? If so, how?

11. Write a recursive (NOW-NEXT) function that would help you make predictions for this data.

12. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the number of 53 year increments in the investigation and let f(x) equal the amount of radioactive substance remaining.

13. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the number of years in the investigation and let f(x) equal the amount of radioactive substance remaining. Use your equation to determine how much radioactive substance will remain after 500 years.

14. When will there be exactly 5 milligrams of the radioactive substance? Determine your answer to the nearest month.

15. Compare Investigation 1 and Investigation 2. What are the similarities and differences?

Investigation 3:
You invest money in a savings account that earns 2.5% interest annually. How much money will you have at the end of 10 years if you begin with $1000?

Complete the table.

| Years | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Money in your account | | | | | | | | | | | |

Questions:

16. What is your initial value for this set of data? What does it represent in the investigation?

17. Would it make more sense to find a common ratio (r) or common difference (d) for this data? Explain.

18. Based on your answer to Question 17, find the r OR d for the data you collected. Show the process you used to do so.

19. Could you estimate your answer to Question 18 without filling in the table? If so, how?

20. Write a recursive (NOW-NEXT) function that would help you make predictions for this data.

21. Write an explicit function using function notation that would help you make predictions for this data. In your function let x be the number of years in the investigation and let f(x) equal the amount of money in the account.

22. You find a bank that will pay you 3% interest annually, so you consider moving your account. Your current bank decides you're a good customer and offers you a special opportunity to compound your interest semiannually!!! (They make it sound like it's a really good deal, so you're curious). You don't play around with your money, so you ask what exactly that means. They explain that you'll still get 2.5% interest, but they'll give you 1.25% interest at the end of June and 1% interest at the end of December. You want to see if you make more money than you would if you just switched banks, so you do the calculations. Which bank is giving you a better deal? Explain your answer.

When you write an equation for a situation and use it to make predictions, you assume that other people who use it will understand the situation as well as you do. That's not always the case when you take away the context, so we sometimes need to provide some additional information to accompany the equation.

23. The domain of a function is the set of all the possible input values that can be used when evaluating it. If you remove your functions in Questions 6 and 13 and 21 from the context of this situation and simply look at the table and/or graph of the function, what numbers are part of the theoretical domain of the function?

    Will this be the case with all exponential functions? Why or why not?

24. When you consider the context, however, not all of the numbers in the theoretical domain really make sense. We call the numbers in the theoretical domain that make sense in our situation the practical domain. For instance, in the first investigation, our input values are "Stages". If you look at the tables you created, what numbers would be a part of the practical domain for these investigations?

    Investigation 1

    Investigation 2

    Investigation 3

    Make note of any similarities and differences and explain why they exist.

# Guided Practice : Exponential Growth & Decay

1. You decide to conduct an investigation in a room of 100 standing students who were to randomly choose a number between and including 1 through 20. If they chose a multiple of 4, they were to sit down. You record the number of people still standing after each turn.
   a. What is the probability of choosing a multiple of 4?
   b. What is the common ratio for this investigation?
   c. What is the initial value of this investigation?
   d. Write a recursive equation for the investigation.
   e. Write an explicit equation for the investigation.

2. Create an investigation which could be modeled by the following equation $f(x) = 33\left(\frac{1}{3}\right)^x$.

3. The amount of radioactive ore in a sample can be modeled by the equation $y = 20(0.997)^x$, where x represents years and y is the amount of ore remaining in milligrams.
   a. What was the initial amount of radioactive ore?
   b. Is this an example of exponential growth or decay?
   c. What percentage of the ore is being lost or gained according to this model?
   d. When will there be half of the initial amount of the radioactive ore?

4. Complete the following table.

| Explicit Function | Recursive Function | Initial Value | Common Ratio | Growth or Decay? |
|---|---|---|---|---|
| $y = 2(3)^x$ | | | | |
| | $NEXT = NOW \cdot 0.5$, $Start\ at - 3$ | | | |
| | | 0.125 | 4 | |
| $y = ab^x$ | | | | growth |

5. The height of a plant can be modeled by the table below.

| Day | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Height (in) | 2.56 | 6.4 | 16 | 40 | 100 |

a) What is the initial height of the plant? Explain how you found your answer?

b) What is the common ratio? Explain how you found your answer.

c) Is this an example of exponential growth or decay? How could you find the answer if you only had the common ratio?

d) Write the recursive function for this situation.

e) Write the explicit function for this situation.

f) How tall will the plant be on the $12^{th}$ day of data collection? How tall is this in inches, feet, yards, and miles? Does this seem realistic to you?

g) On what day will the plant first be over 100 yards tall?

6. When you opened a savings account on your $15^{th}$ birthday, you deposited most of the money from your summer job ($2000). The banker who helped you informed you that you would received 1.5% interest each year.
   a. How much money will you have in the account when you turn 21?

   b. Use the properties of exponents to determine the ***monthly*** percentage interest rate that you could have gotten from the bank that would have given you the same amount of money when you turned 21.

7. For each scenario, write an explicit equation, define your variables, and determine the practical and theoretical domain.

a) The town of Braeford was first established in 1854 when it had a population of 24. Since then it has grown by a percentage of 1.25% each year.

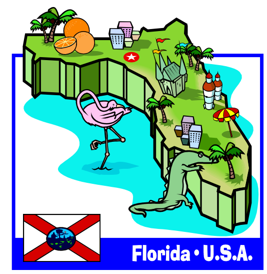
b) A species of bacteria reproduces exactly once each hour on the hour. At this exact time, each organism present divides into two organisms. One of these bacteria is placed into a petri-dish at 8:00 this morning.

# Practice : Exponential Growth & Decay

## 1POPULATION

In 1990, Florida's population was about 13 million. Since1990, the state's population has grown about 1.7% each year. This means that Florida's population is growing exponentially.

| Year | Population |
|---|---|
| 1990 | |
| 1991 | |
| 1992 | |
| 1993 | |
| 1994 | |




a) Write an explicit function in the form $y = ab^x$ that models the values in the table.

b) What does x represent in your function?

c) What is the "a" value in the equation and what does it represent in this context?

d) What is the "b" value in the equation and what does it represent in this context?

## 2 HEALTHCARE

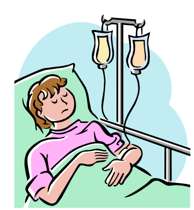
Since 1985, the daily cost of patient care in community hospitals in the United States has increased about 8.1% per year. In 1985, such hospital costs were an average of $460 per day.

a) Write an equation to model the cost of hospital care. Let x = the number of years after 1985.

b) Find the approximate cost per day in 2012.

c) When was the cost per day $1000

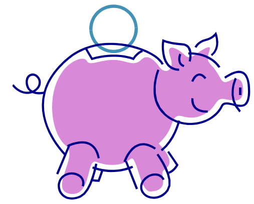
d) When was the cost per day $2000?



## 3HALF-LIFE

To treat some forms of cancer, doctors use Iodine-131 which has a half-life of 8 days. If a patient received 12 millicuries of Iodine-131, how much of the substance will remain in the patient 2 weeks later?

**4 SAVINGS**



Suppose your parents deposited $1500 in an account paying 6.5% interest compounded annually when you were born.

a) Find the account balance after 18 years.

b) What would be the difference in the balance after 18 years if the interest rate in the original problem was 8% instead of 6.5%?

c) What would be the difference in the balance if the interest was 6.5% and was compounded monthly instead of annually.

**5 HEALTH**

Since 1980, the number of gallons of whole milk each person in the US drinks in a year has decreased 4.1% each year. In1980, each person drank an average of 16.5 gallons of whole milk per year.

| Year | Population |
|---|---|
| 1980 | |
| 1981 | |
| 1982 | |
| 1983 | |
| 1984 | |

a) Write a recursive function for the data in the table.

b) Write an explicit function in the form $y = ab^x$ that models the values in the table. Define your variables.

c) According to this same trend, how many gallons of milk did a person drink in a year in 1970?

**6 WASHINGTON, D.C.**


The model $y = 604000(0.982)^x$ represents the population in Washington, D.C. $x$ years after 1990.

a) How many people were there in 1990?

b) What percentage growth or decay does this model imply?

c) Write a recursive function to represent the same model as the provided explicit function.

d) Suppose the current trend continues, predict the number of people in DC now.

e) Suppose the current trend continues, when will the population of DC be approximately half what it was in 1990?