

Simplify the following exponents

1. $(14a^4b^6)^2 (a^6c^3)^7 = 14^2 a^8 b^{12} \cdot a^{42} \cdot c^{21}$
 $196 a^{50} b^{12} c^{21}$

3. $\left(\frac{x^{-8}}{y^{11}}\right)^{-2} = \frac{x^{16}}{y^{-22}} = x^{16} y^{22}$

2. $\left(\frac{4d^3}{c^5}\right)^3 = \frac{4^3 d^9}{c^{15}} = \frac{64d^9}{c^{15}}$

4. $(g^3 \cdot g^{-2})^4 = (g^1)^4 = g^4$

Simplify each expression, and write your final answer with exponents.

*Hint: Change all radicals to exponents first!

5. $\sqrt[3]{k} \cdot k^{6/4} = k^{1/3} \cdot k^{3/2}$
 $k^{\frac{1}{3} + \frac{3}{2}} = k^{\frac{10}{6}} = k^{5/3}$

6. $\sqrt{36s^2} \cdot (s^6)^{1/3}$
 $(36s^2)^{1/2} \cdot s^2$
 $6s \cdot s^2 = 6s^3$

7. $2k^{2/3} \cdot \frac{1}{4} k^{5/6}$
 $\frac{1}{2} k^{2/3 + 5/6} = \frac{1}{2} k^{9/6} = \frac{1}{2} k^{3/2}$

Simplify each expression, and write your final answer in simplest radical form.

8. $m^{1/2} \cdot m^{4/3} = m^{1/2 + 4/3} = m^{11/6}$
 $= m^{11/6} \rightarrow \sqrt[6]{m^{11}} \text{ or } (\sqrt[6]{m})^{11}$

9. $\sqrt[4]{256x^8} \cdot \sqrt{8x^3} = (256x^8)^{1/4} \cdot (8x^3)^{1/2}$
 $256^{1/4} x^2 \cdot 8^{1/2} x^{3/2}$
 $4x^2 \cdot 2x^{3/2}$
 $4x^2 \cdot 2^{3/2} x^{3/2}$
 $4x^2 \cdot (2x)^{3/2} = 4x^2 \sqrt[3]{2x}$

Several different acceptable answers

Solve the following equations. Remember to rewrite radicals as exponential expressions and check your solutions.

10. $\sqrt[3]{2x-4} = -2$
 $(2x-4)^{1/3} = -2$
 $2x-4 = (-2)^3 = -8$
 $2x = -4$
 $x = -2$ ✓

11. $\sqrt{x+1} = x+1$
 $(x+1)^{1/2} = x+1$
 $x+1 = (x+1)^2$
 $x+1 = x^2+2x+1$
 $0 = x^2+x$
 $0 = x(x+1)$
 $x=0 \quad x=-1$

$x=0$:
 $\sqrt{0+1} = 0+1$
 $1 = 1$ ✓

$x=-1$:
 $\sqrt{-1+1} = -1+1$
 $0 = 0$ ✓

12. $\sqrt{x-7} = -9$
 $x-7 = 81$
 ~~$x = 88$~~
 extraneous

$\sqrt{88-7} = -9$
 $\sqrt{81} = -9$
 $9 = -9$ No

13. $\sqrt{3x+19} = x-3$
 $(3x+19)^{1/2} = x-3$
 $3x+19 = (x-3)^2$
 $3x+19 = x^2-6x+9$
 $0 = x^2-9x-10$
 $0 = (x-10)(x+1)$
 $x=10 \quad x=-1$

$x=10$:
 $\sqrt{30+19} = 10-3$
 $7 = 7$ ✓

~~$x=-1$~~ extraneous
 $\sqrt{-3+19} = -1-3$
 ~~$4 = 4$~~

Solve the following word problems

14. The function ~~$y = 187900(1.025)^x$~~ represents the value of a home ~~x~~ years after purchase. Find the monthly and quarterly rate of appreciation of the home.

SKIP

15. In a dish, there are 18 bacteria. Ten hours later, there are 180 bacteria in the dish. $P(t) = 18(10^{0.1t})$ provides an exponential growth model that matches these data points.

a. Find the amount after 4 hours.

$$18 \cdot (10^{0.4}) = 45.21 \rightarrow \text{45 bacteria}$$

b. Use the given function to estimate the time when the bacteria would be 20,000.

Key:
 $\log x^a = a \log x$

$$20000 = 18 \cdot (10)^{.1t}$$

$$\frac{20000}{18} = (10)^{.1t}$$

$$\log \frac{20000}{18} = .1t \log 10$$

$$\log \frac{20000}{18} = .1t$$

$$t = 10 \left(\frac{\log \frac{20000}{18}}{\log 10} \right) = \text{30.46 hours}$$

16. The buffalo population in the Midwest is decreasing by a rate of 15% each year. The population in 1904 is 200. Write an explicit equation to model this situation. In what year will the population be 500?

$$500 = 200 (.85)^x$$

$$2.5 = (.85)^x$$

$$x = \frac{\log 2.5}{\log (.85)}$$

$$x = -5.6$$

In 1898 it was 500.

incorrect

Convert from exponential to log form or log to exponential form

17. $\left(\frac{2}{3}\right)^x = \frac{1}{9}$

$$\log_{\frac{2}{3}} \left(\frac{1}{9}\right) = x$$

18. $3^x = 42$

$$\log_3 42 = x$$

19. $\log_2 p = q$

$$2^q = p$$

20. $\log_u \frac{1}{4} = v$

$$u^v = \frac{1}{4}$$

Solve for the variables in the exponents

21. $(2^{x+1})^5 = 2^x$

$$2^{5x+5} = 2^x$$

$$5x+5 = x$$

$$4x+5=0$$

$$4x = -5$$

$$x = \frac{-5}{4} \text{ or } -1.25$$

22. $10^{2x} - 3 = 997$

$$10^{2x} = 1000$$

$$10^{2x} = 10^3$$

$$2x = 3$$

$$x = \frac{3}{2} \text{ or } 1.5$$

23. $2(3^{2x-5}) = 86$

$$3^{2x-5} = 43$$

$$(2x-5) \log 3 = \log 43$$

$$2x-5 = \frac{\log 43}{\log 3}$$

$$2x = \frac{\log 43}{\log 3} + 5$$

$$x = \frac{1}{2} \left(\frac{\log 43}{\log 3} + 5 \right)$$

$$x = 4.21$$

24. $3(10)^{x+4} + 3 = 15$

$$3(10)^{x+4} = 12$$

$$(10)^{x+4} = 4$$

$$(x+4) \log 10 = \log 4$$

$$x+4 = \frac{\log 4}{\log 10}$$

$$x = \frac{\log 4}{\log 10} - 4$$

$$x = -3.4$$

$$y = \frac{25^{\frac{1}{2}x}}{9^{\frac{1}{2}x}}$$

$$y = \frac{5^x}{3}$$

Fill in the blank to make each statement true

Replace blank with y and solve for y

25. $(\quad)^3 = 8x^3$

$$y^3 = 8x^3$$

$$y = (8x^3)^{\frac{1}{3}} = 2x$$

26. $5x^3 \cdot \quad = 125x$

$$5x^3 \cdot y = 125x$$

$$y = \frac{125x}{5x^3} = \frac{25}{x^2}$$

27. $(\quad)^{-2} = (9/25x^2)$

$$y^{-2} = \frac{9}{25x^2}$$

$$\frac{1}{y^2} = \frac{9}{25x^2}$$

$$y^2 = \frac{25x^2}{9}$$

$$y = \frac{5x}{3}$$