#### Name:

## Graphing Advanced Functions

This unit will get into the graphs of simple rational (inverse variation), radical (square and cube root), piecewise, step, and absolute value functions. You should continue with using transformations to help you graph from a parent function. We will use function notation throughout and use it to model and evaluate simple power functions and inverse variation. The unit will also dig into how we solve simple rational (inverse variation) and radical (square root and cube root) equations and introduce the idea of extraneous solutions. We will also look into solving systems of equations with linear and inverse variations. In this unit, students will ...

- Graph a function and its translation. (F-BF.3)
- Identify how the graph of a function has changed from its parent function. (Honors: discuss the order of transformations given that multiples can occur in a function. (**F-BF.3**)
- Use function notation. (F-IF.2)
- Analyze a function and its graph based on its key features. (Honors: Range and asymptotes are discussed and change with translations) (F-IF.4)
- Solve simple rational equation (Honors: extend to rationals with linear and factorable quadratic terms) (A-REI.2)
- Solve radical equations (A-REI.2)
- Model situations using inverse variation (F-BF.1)
- Explain why a solution is extraneous and give examples of extraneous solutions (A-REI.2)
- Create equations and inequalities in one variable (A-CED.1)
- Use equations and inequalities to solve problems. (A-REI.2)
- Represent constraints by equations or inequalities. (A-CED.3)

Day	Activity
Monday, 4/18	Domain & Range Pictionary
	Start Fred Functions
Tuesday, 4/19	Finish Fred Functions
	Quadratic&Absolute Value Transformations
Wednesday, 4/20	Graphing 2^x and log(x)
Thursday, $4/21$	Graphing Square and Cube Root
Friday, 4/22	Review & Quiz
Monday, 4/25	Graphing Inverse Variation
Tuesday, 4/26	Graphing Step Functions
Wednesday, 4/27	Graphing Piecewise Functions Intro
Thursday, $4/28$	Review & Quiz
Friday, 4/29	Graphing Piecewise Practice & Review
Monday, 5/2	Unit 6 Test



To the right is a graph of a "piece-wise" function. We'll call this function <b>F(x)</b> . We can use <b>F(x)</b> to	
explore transformations in the coordinate plane.	
<ol> <li>How do we know that F(x) is a function? (Hint: How do we define a function?)</li> </ol>	
2. What is the domain of <b>F(x)</b> ?	
3. What is the range of <b>F(x)</b> ?	
Let's expl	ore the points on F(x).
<ul><li>4. How many points lie on F(x)? Can we list them all?</li></ul>	
<ol> <li>What are the key points that would help us graph F(x)?</li> </ol>	
<i>We are will call these key points <b>"characteristic" points</b>. It is important when graphing a function that you are able to identify these characteristic points.</i>	
<ul><li>6. Use the graph of F(x) to evaluate the following:</li></ul>	$F(1) = \_$ $F(-1) = \_$ $F(5) = \_$
*Remember that F(x)	is another name for the y-values*
7. Fill the three tables using the graph of <b>F(x)</b> .	x $F(x) + 4$ x $F(x) - 3$
$\begin{array}{ c c c } \hline x & F(x) \\ \hline -1 & \end{array}$	-1 -1
	1
	1     1       2     2
2       4       8. Graph F(x) + 4 and F(x) - 3 in different colors on the coordinate plane above	1     2       2     2       4     4
2 4 8. Graph <b>F(x) + 4</b> and <b>F(x) - 3</b> in different colors	1     2       2     2       4     4

11. Fill the three tables using the graph of <b>F(x)</b> .	x	x + 4	y = F(x +				
x F(x)	-5	-1	<b>4)</b> 1			st box we ha ve subtract 4	
-1	-5				both side	es of the get $x = -5$ . Us	se a
1		1	-1	simila	r metho	d to find the	
2		2	-1	remai	ining x va	alues.	
4		4	-2		(	y = F(x -	٦
				х	(x - 3)	y = F(x - 3)	
					-1	1	
					1	-1	
					2	-1	
					4	-2	
12. On the coordinate plane to the right:							
a. Use one color to graph the 4 ordered							
pairs $(x, y)$ for $y = F(x + 4)$ . The first point is (-5, 1).							
point is (-3, 1).							
b. Use a different color to graph the 4			F(x)				
ordered pairs $(x, y)$ for $y = F(x - 3)$ .							
				┨┤┤┤			
13. In <b>y</b> = $F(x+4)$ , how did the "+4" affect the							
graph of <b>F(x)</b> ? What type of <i>transformation</i>							
maps $F(x)$ to $F(x + 4)$ ? (Be specific)							
14. In $\mathbf{y} = \mathbf{F}(\mathbf{x} - 3)$ , how did the "-3" affect the							
graph of <b>F(x)</b> ? What type of <i>transformation</i> maps <b>F(x)</b> to <b>F(x – 3</b> )? (Be specific)							
15. Fill the tables using the graph of $F(x)$ .	x	-F(x)	x	2F(x)		x ½F(x)	_
x F(x)	-1		-1			-1	
	1		1			1	
	2					2	-
2			2		-		_
4	4		4			4	

16. How did each of the following affect the graph of <b>F(x)</b> :	<i>Hint: Use one of the coordinate planes above if needed.</i> a)
a) the "-" sign	h
b) the "2"	b)
c) the "½"	c)

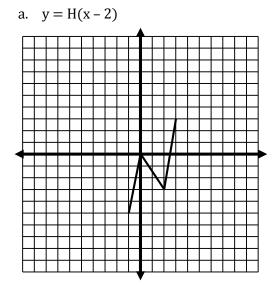
Summary: Describe the effect to F(x) for the following functions.

Equation	Effect on the graph of F(x)
Example: $y=F(x+18)$	Translate <b>F(x)</b> to the left 18 units
1. $y = F(x) - 100$	
2. $y = F(x - 48)$	
3. $y = F(x) + 32$	
4.  y = -F(x)	
5. $y = F(x - 10)$	
6. $y = F(x) + 7$	
7. $y = \frac{1}{4}F(-x)$	
8. $y = F(x) - 521$	
9. $y = F(x) + 73$	
10. $y = -5F(x)$	
11. $y = F(x) - 22$	
12. $y = 2F(x - 13)$	
13. $y = F(x + 30) + 18$	
14. y = $-\frac{1}{4}F(\frac{1}{3}x) - 27$	

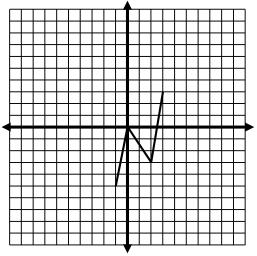
To the right is a graph of a "piece-wise" function that we'll call **H(x)**.

Use **H(x)** to demonstrate what you have learned so far about the transformations of functions.

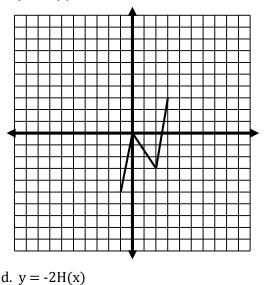
- 1. What are the characteristic points of **H(x)**?
- 2. Describe the effect on the graph of H(x) for each of the following:
  - a. H(x 2)
  - b. H(x) + 7
  - c. H(x+2) 3
  - d. -2H(x)
- 3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

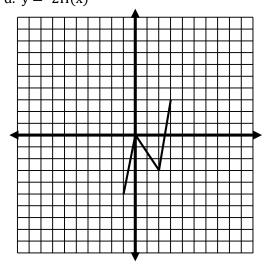


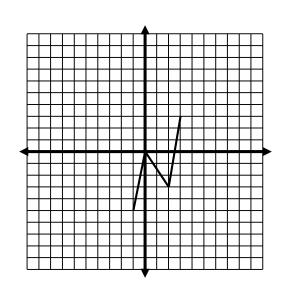
c. y = H(x+2) - 3



b. y = H(x) + 7

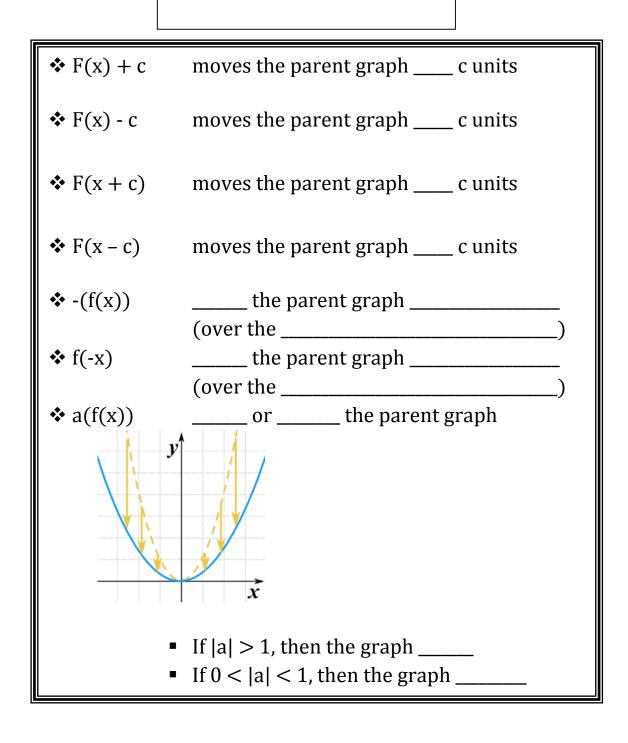






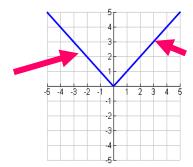
## **Graphing Quadratic Functions**

- A quadratic graph is in a U shape called a \_\_\_\_\_
- Quadratic graphs follow the same rules as "Fred"
- Quadratic functions have a \_\_\_\_\_\_ that can be found by identifying the \_\_\_\_\_\_ and \_\_\_\_\_\_ transformations.
- Quadratic functions that have been transformed are in the form:

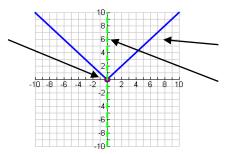


### **Graphing Absolute Value Functions**

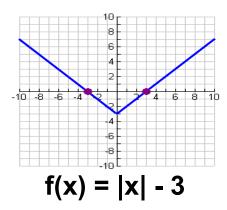
- The function f(x) = |x| is an \_\_\_\_\_
- The graph of this piecewise function consists of 2 rays, is v-shaped, and opens up.



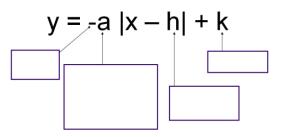
- The highest or lowest point on the graph of an absolute value function is called the \_\_\_\_\_\_.
- An \_\_\_\_\_\_ of the graph of a function is a vertical line that divides the graph into mirror images.
  - An absolute value graph has \_\_\_\_\_\_ axis of symmetry that passes through the \_\_\_\_\_\_.



- The \_\_\_\_\_\_ of a function f(x) are the values of x that make the value of f(x) zero.
- On this graph where \_\_\_\_\_\_ and \_\_\_\_\_ are where the function would equal 0.

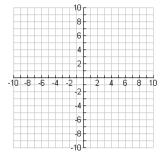


- A \_\_\_\_\_\_ changes a graph's size, shape, position, or orientation.
- A \_\_\_\_\_\_ is a transformation that shifts a graph horizontally and/or vertically, but does not change its size, shape, or orientation.
- A \_\_\_\_\_\_ is when a graph is flipped over a line. A graph flips \_\_\_\_\_\_ when -1. f(x) and it flips \_\_\_\_\_\_ when f(-1x).
- A \_\_\_\_\_\_ changes the size of a graph by stretching or compressing it. This happens when you \_\_\_\_\_\_ the function by a number.

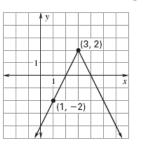


\*Remember that (h, k) is your vertex\*

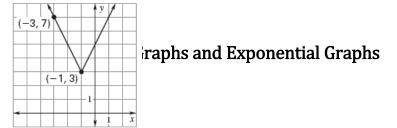
- Example 1: Identify the transformations:
  1. y = 3 |x + 2| 3
  - 2. y = |x 1| + 2
  - 3. y = 2 |x + 3| 1
  - 4. y = -1/3|x 2| + 1
- Example 2: Graph y = -2 |x + 3| + 2.
  What is your vertex?
  - What are the intercepts?
  - What are the zeros?

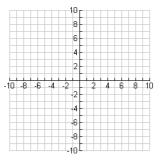


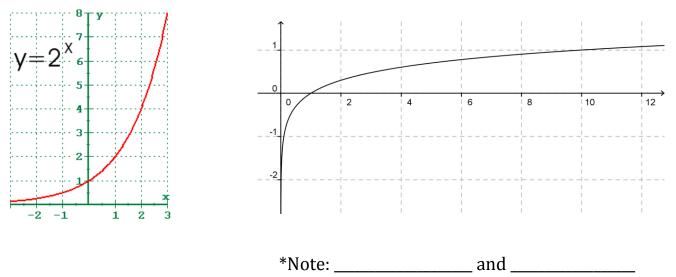
- You Try: Graph y = -1/2 |x 1| 2
  - Compare the graph with the graph of y = |x| (what are the transformations)
- **Example 3**: Write a function for the graph shown.



• You Try: Write a function for the graph shown.







are the same graph

# Define Asymptote:

Key Features of Exponential and Logarithmic Functions				
Characteristic	Exponential	Logarithmic		
	Function	Function		
	$y=2^{x}$	$y = \log x$		
Asymptote				
Domain				
Range				
Intercept				

Translations of logarithmic functions are very similar to those for other functions and are summarized in the table below.

Parent Function	$y = \log x$	Y=2 <sup>x</sup>
Shift up		
Shift down		
Shift left		
Shift right		
Combination Shift		
Reflect over the x-axis		
Stretch vertically		
Stretch horizontally		

Let's look at the following example.

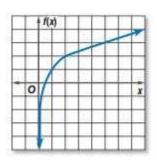
The graph on the right represents a transformation of the graph of  $f(x) = 3 \log_{10} x + 1$ .



- h = 0: \_\_\_\_\_.
- k = 1: \_\_\_\_\_.

Domain:

Range:



Asymptote:

# TRY NOW

Graph the following function on the graph at right. Describe each transformation, give the domain and range, and identify any asymptotes.

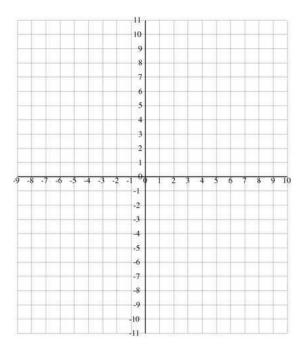
 $y = -2log_{10}(x + 2) - 4$ 

Domain:

Range:

Asymptote:

Description of transformations:



#### **Guided Practice with Logarithmic Functions**

Graph the following transformations of the function  $y = \log_{10} x$  on the coordinate planes. Determine the domain, range, and asymptotes of each transformation. Describe the transformations.

1) $y = \log_{10} x - 6$	2) $y = -\log_{10} (x + 2)$	3) $y = \log_{10} 2x$
Domain:	Domain:	Domain:
Range:	Range:	Range:
Asymptotes:	Asymptotes:	Asymptotes:
Description:	Description:	Description:

Graph the following transformations of the function  $y = 2^x$  on the coordinate planes. Determine the domain, range, and asymptotes of each transformation. Describe the transformations.

4) $f(x) = 2^{x+1} - 3$	5) $f(x) = -2^{x} - 1$	6) $f(x) = 2^{x-5} + 2$
Domain:	Domain:	Domain:
Range:	Range:	Range:
Asymptotes:	Asymptotes:	Asymptotes:
Description:	Description:	Description:

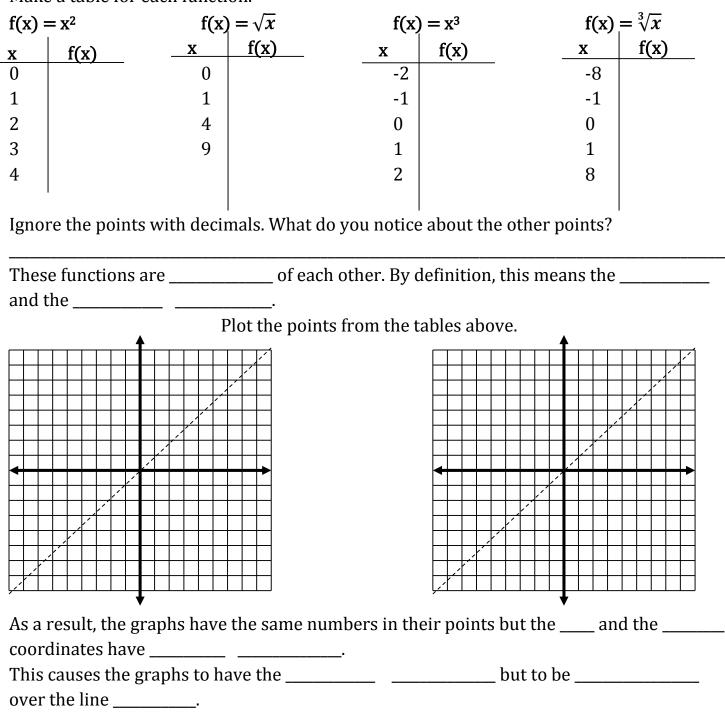
### 11

7) State the domain, range, intercepts and asymptotes of  $f(x) = \log(x - 2) + 3$ .

8) Describe the transformations of  $y = 4 \log (2x - 4) + 6$  from the parent function  $y = \log(x)$ .

9) Describe the transformations of  $y = -3 \log_{10} (4x + 3) - 2$  from the parent function  $y = \log_{10} (x)$ .

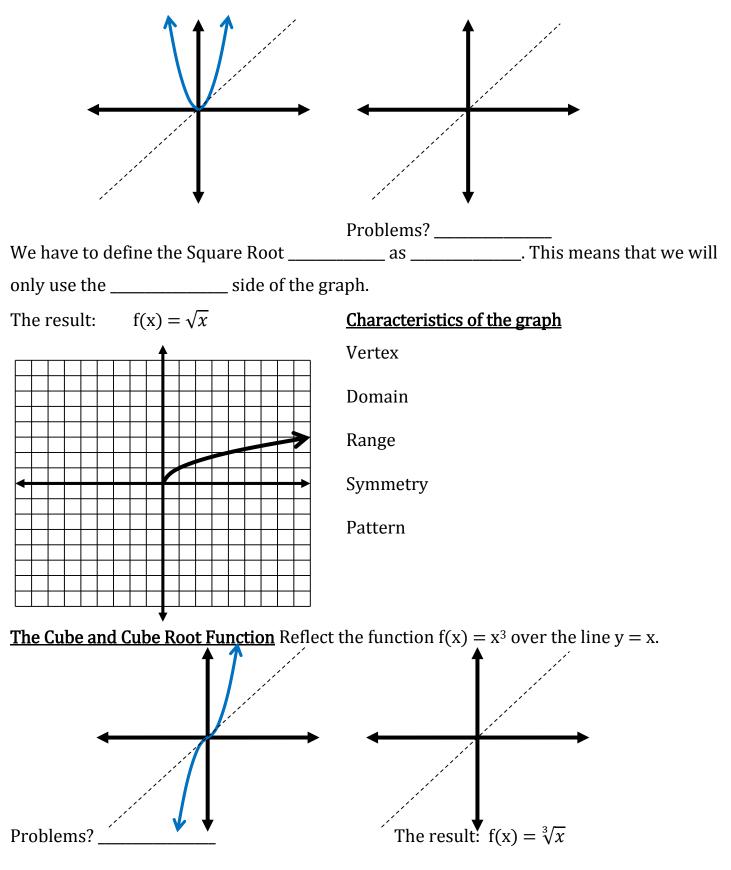
## **Graphing Square and Cube Root Functions**

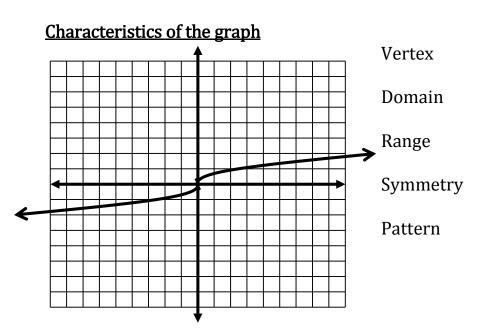


Make a table for each function.

# The Square and Square Root Function

Reflect the function  $f(x) = x^2$  over the line y = x.





## **Transforming the Graphs**

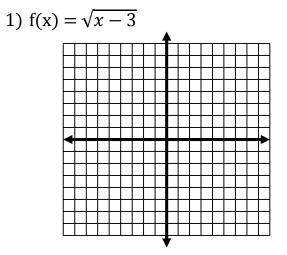
Now that we know the shapes we can use what we know about transformations to put that shape on the coordinate plane.

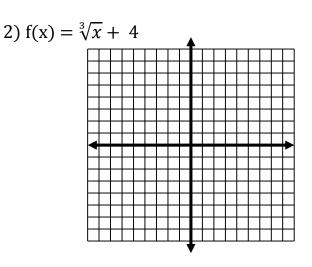
Remember:

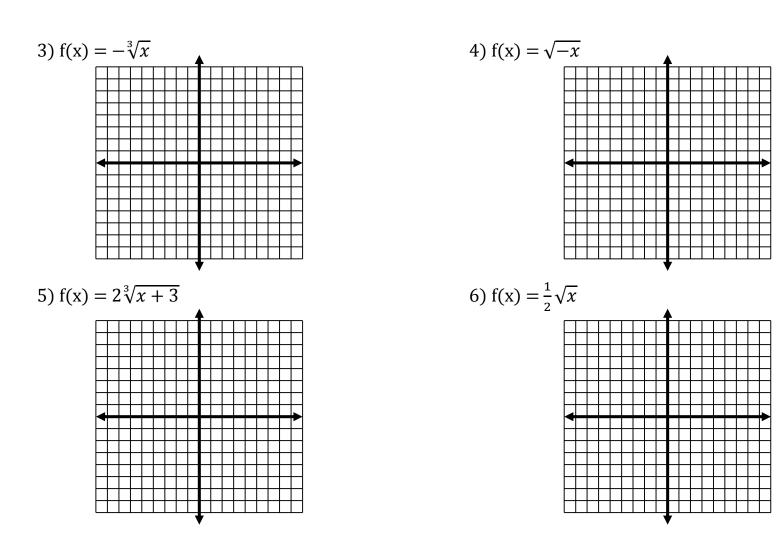
<u>Translate</u>

<u>Reflect</u>

<u>Dilate</u>







Sometimes the functions are not in graphing form. We may have to use some of our algebra skills to transform the equations into something we can use.

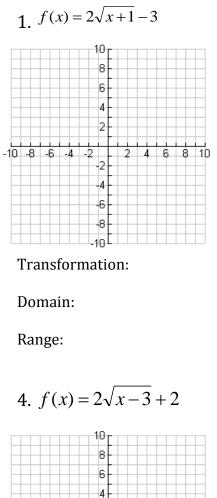
Ex:  $f(x) = \sqrt{4x - 12}$ 

This is not in graphing form.

Ex:  $f(x) = \sqrt[3]{8x + 32} - 5$ 

This is not in graphing form.

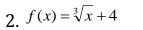
# Graph each function, and identify its domain and range. For 5-6 put in graphing form first.

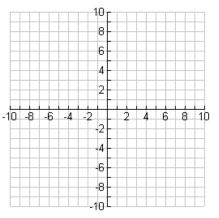


Transformation:

Domain:

Range:



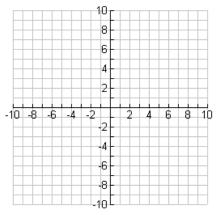


Transformation:

Domain:

Range:

$$5. f(x) = \sqrt[3]{8x + 16} - 3$$

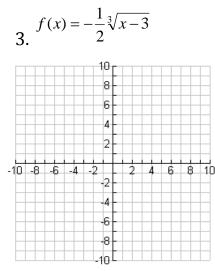


Graphing Form:

Transformation:

Domain:

Range:

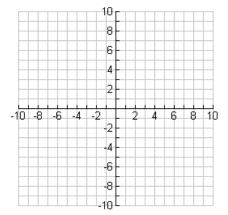


Transformation:

Domain:

Range:

6. 
$$f(x) = \sqrt{9x - 27} - 2$$



Graphing Form:

Transformation:

Domain:

Range:

# **Graphing Inverse Variation**

The constant <i>k</i> is the		$\frac{k}{x}$ , where <i>k</i> is a nonzero constant and $x \neq 0$ , is an
(the inverse, or opposite, of increase). The domain is all real numbers The range is all real numbers Why? Why? Since both the domain and range have restrictions at zero, the graph can never touch the x and y axis. This creates asymptotes The graphs of inverse variations have two parts. Ex: $f(x) = \frac{1}{x}$ Each part is called a When k is, the branches are in Quadrants and When k is, the branches are in Quadrants and Translations of Inverse Variations: The graph of y= is a translation of b units and c units K tells us how far the branches have been from the We can use it to help	The constant <i>k</i> is the	
Why?       Why?         Since both the domain and range have restrictions at zero, the graph can never touch the x and y axis.       This creates asymptotes		while the other quantity will
Since both the domain and range have restrictions at zero, the graph can never touch the x and y axis. This creates asymptotes The graphs of inverse variations have two parts. Ex: $f(x) = \frac{1}{x}$ Each part is called a When k is, the branches are in Quadrants and When k is, the branches are in Quadrants and When k is, the branches are in Quadrants and Translations of Inverse Variations: The graph of y= is a translation of, b units and c units t tells us how far the branches have been from the We can use it to help	The domain is all real numbers	The range is all real numbers
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The graphs of inverse variations have two parts. Ex: $f(x) = \frac{1}{x}$ Each part is called a When k is, the branches are in Quadrants and When k is, the branches are in Quadrants and Translations of Inverse Variations: The graph of y= is a translation of, b units and c units The vertical asymptote is The horizontal asymptote is k tells us how far the branches have been from the We can use it to help	Since both the domain and range have restrictions	s at zero, the graph can never touch the x and y axis.
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The vertical asymptote is The horizontal asymptote is <i>k</i> tells us how far the branches have been from the We can use it to help		
<i>k</i> tells us how far the branches have been from the We can use it to help	is a translation of, b units and c	c units
	The vertical asymptote is The horizo	ontal asymptote is

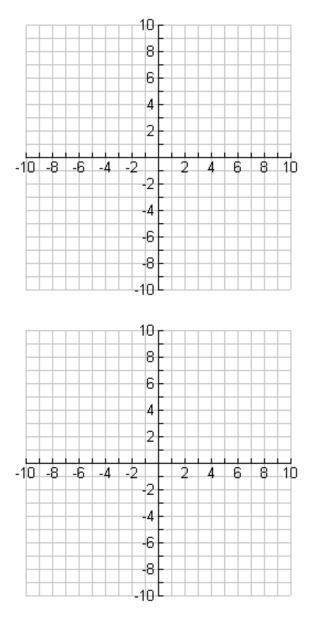
**Example 1:** 
$$y = \frac{1}{x-3} + 4$$

Vertical Asymptote:

Horizontal Asymptote:

Quadrants:

Distance from the Asymptote:



**You Try:**  $y = -\frac{4}{x+1}$ 

Vertical Asymptote:

Horizontal Asymptote:

Quadrants:

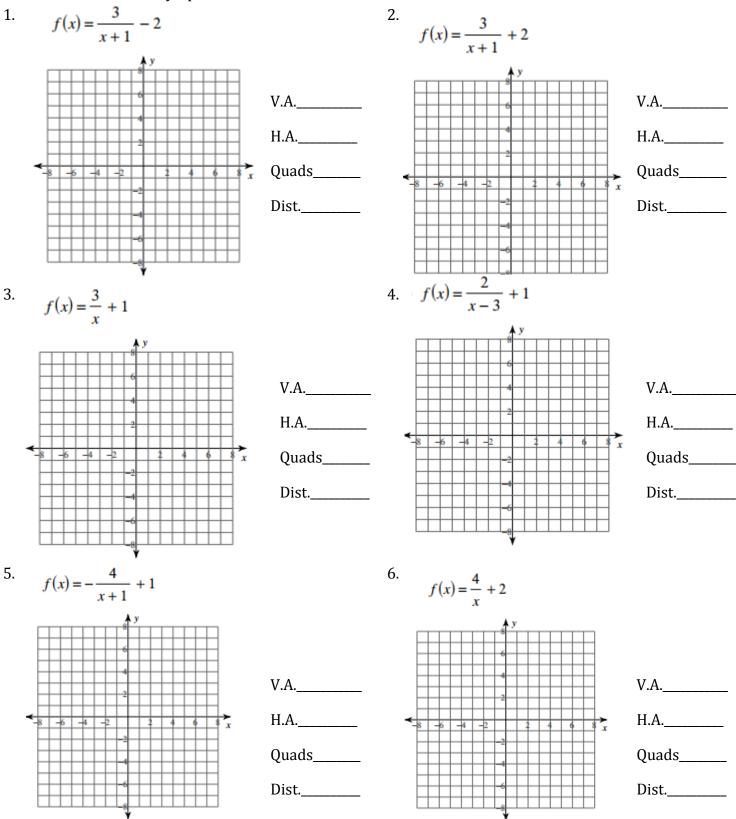
Distance from the Asymptote:

We can also write the equation just given the parent function and the asymptotes.

**Example:** Write the equation of  $y = -\frac{1}{x}$  that has asymptotes x = -4 and y = 5.

#### **Graphing Inverse Variation Practice**

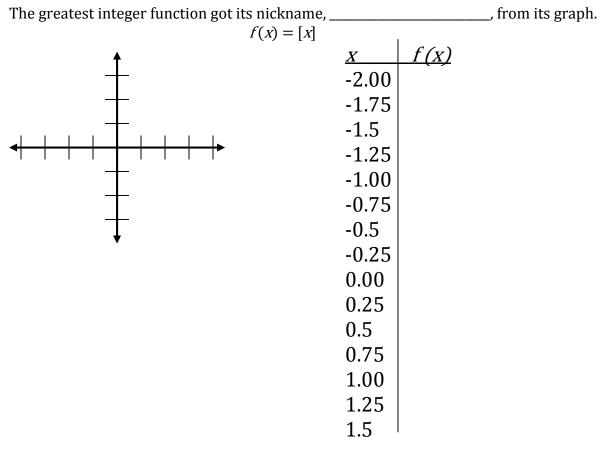
For each of the graphs, identify the horizontal and vertical asymptotes, quadrants where it is located and the distance from the asymptotes.



7. Write the equation of  $y = -\frac{1}{x}$  that has asymptotes x = 2 and y = 4 that is 3 units from the asymptotes.

# **Greatest Integer Function**

The Greatest Int	eger Function			_
f(x) =				
This function tal	xes the input and	finds the		closest to that
number				
Examples:	Answers	Examples:	Answers:	
1. [7.35] =		3. [-2.5]		
$2.\left[\frac{4}{3}\right] =$		$4. \left[-\frac{10}{5}\right]$		
Graphing the Great	est Integer Function			



## **Transformations of the Greatest Integer Function**

Don't forget the transformations do not change!

Graphing Form: \_\_\_\_\_

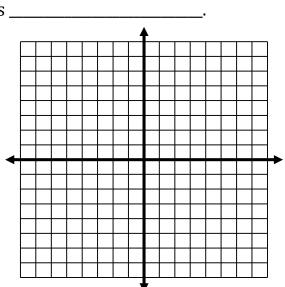
- \_\_\_\_\_ is a starting point for your steps.
- The length of your steps is \_\_\_\_\_\_.
- The space between your steps (vertically) is \_\_\_\_\_\_.
- If \_\_\_\_\_ is \_\_\_\_\_ the steps go \_\_\_\_\_\_
- If \_\_\_\_\_\_ is \_\_\_\_\_\_\_ then the steps \_\_\_\_\_\_.

**Example 1**: Graph f(x) = 2[[x - 3]] + 1

Start \_\_\_\_\_

Step l	ength	
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Step height \_\_\_\_\_



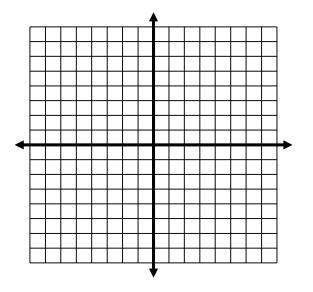
## **Example 2**: Graph y = [[2x + 4]] - 5

Get in graphing form! \_\_\_\_\_

Start \_\_\_\_\_

Step length \_\_\_\_\_

Step height \_\_\_\_\_



[x] is the Greatest Integer Function. It is the largest integers less than or equal to x.

Evaluate the following:

1. [[7.1]]	2. [[1.8]]	3. [[π]]	4. [[-6.8]]
5. [[-2.1]]	6. [[0]]	7. [5.28]	8. <mark>8</mark> ]
9. [[0.25]]	10. <b>[-</b> 0.25]	11. 3[[0.75]]	125 <b>[-</b> 2.5]
13. [[2(1.55)]]	14. 2[[1.55]]	15. 0.5[[1.5]]	16. [[1.25 <b>-</b> 5]]
17. [5 - 1.25]	18. [3(1.25)]	19.3[1.25]	20. [-5]

#### Using a table of values, graph each function.

3. f(x) = [x]

	_	Ц	_			
$\square$	+	$\square$	_			
$\vdash$	+	$\square$	_			
$\vdash$	+	$\vdash$	_			
	$\rightarrow$	$\square$	_			

5. 
$$f(x) = [x] + 3$$

					6. $f(x) = 2[x]$
$\square$	+	H	+		
		H			

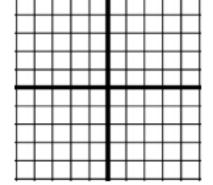
4. 
$$f(x) = [x] + 2$$

		_	_		

$\vdash$								
$\vdash$			-	-				-
$\vdash$	$\vdash$	$\vdash$	$\vdash$	Н	$\vdash$		Н	$\vdash$
								-

8. 
$$f(x) = [-x + 2]$$


7. 
$$f(x) = [x - 3]$$



11. f(x) = -[[x]] + 3

_	_	_	_	_	_	_	_	_	_

13. 
$$f(x) = 2[[x - 3]]$$

14. f(x) = [2x]

		-			
		_			

10. f(x) = [-x]

$\vdash$			$\vdash$		
+					
$\vdash$	H	Η	$\vdash$	$\square$	

12. f(x) = [x + 2]

14. f(x) = -[x + 2]

$\vdash$						
$\vdash$	Η	Η	$\vdash$		$\vdash$	

$\vdash$					
					-

15. f(x) = [[2x]]-3

$\square$					
$\Box$					
$\Box$					
$\square$					
$\square$					

#### **Piecewise Functions**

- In real life functions are represented by a combination of equations, each corresponding to a part of the domain.
- These are called \_\_\_\_\_\_
- One equation gives the value of f(x) \_\_\_\_\_  $f(x) = \begin{cases} 2x 1 & \text{if } x \le 1 \\ 3x + 1 & \text{if } x > 1 \end{cases}$

Example: Evaluate f(x) when x=0, x=2, x=4  $f(x) = \begin{cases} \frac{x+2, \text{ if } x < 2}{2x+1, \text{ if } x \ge 2} \end{cases}$ 

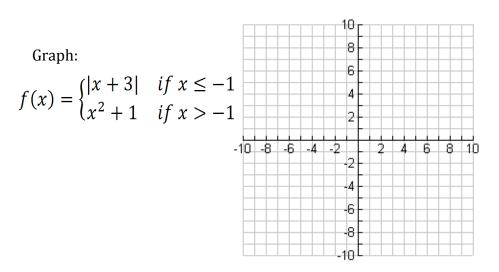
Graph:  $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{if } x < 1 \\ -x + 3, & \text{if } x \ge 1 \end{cases}$ 

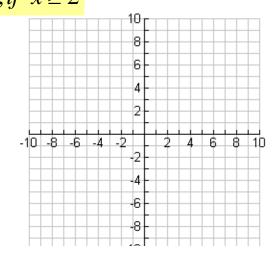
Make a table for each piece. Be sure to select appropriate x vales.

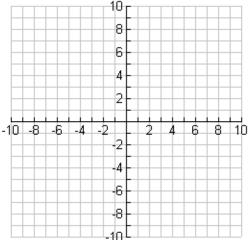
Use \_\_\_\_\_ for < or >

Use \_\_\_\_\_ for  $\leq$  or  $\geq$ 

Graph: 
$$f(x) = \begin{cases} \frac{x-1, if \ x > 2}{-x+1, if \ x \le 2} \end{cases}$$

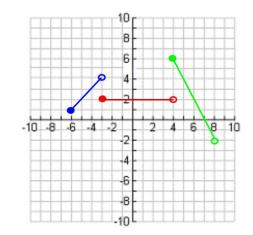






**Lesson 2**: Writing piecewise functions given a graph.

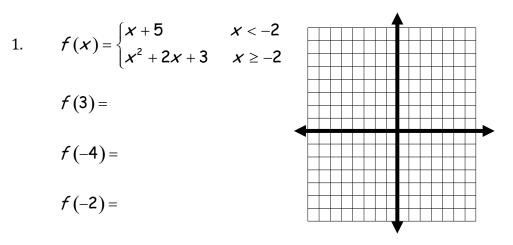
- 3. Can you identify the equations of the lines that contain each segment?
  - a. Left segment equation=
  - b. Middle equation=
  - c. Right equation=
- 4. Next, list the domain of each segment.
  - a. Left segment domain=
  - b. Middle domain=
  - c. Right domain=

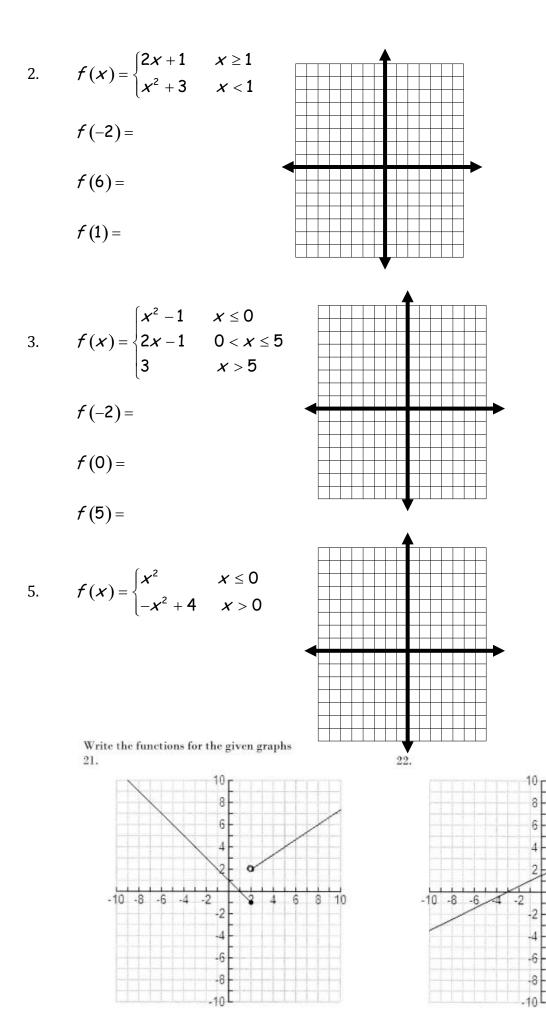


5. Now, put the domain together with the equations to write the piecewise function for the graph.



#### **Piecewise Functions Practice**



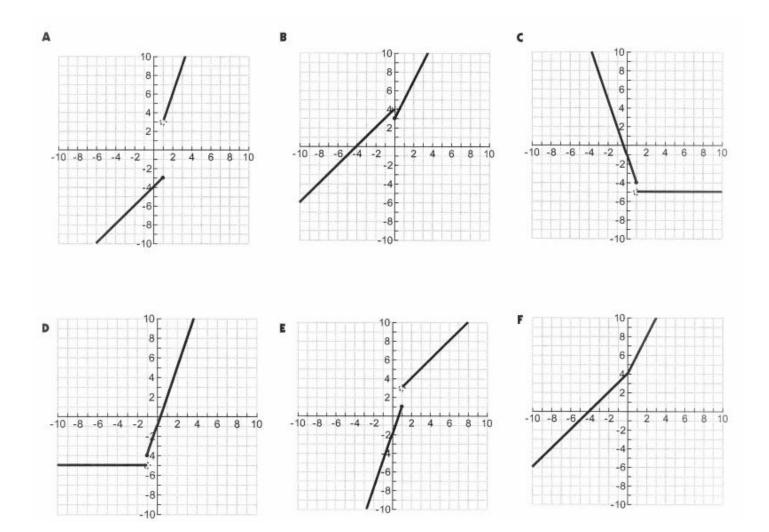


8 10

Match the piecewise function with its graph.

15. 
$$f(x) = \begin{cases} x-4, & \text{if } x \le 1 \\ 3x, & x > 1 \end{cases}$$
 16. 
$$f(x) = \begin{cases} x+4, & \text{if } x \le 0 \\ 2x+4, & \text{if } x > 0 \end{cases}$$
 17. 
$$f(x) = \begin{cases} 3x-2, & x \le 1 \\ x+2 & x > 1 \end{cases}$$

18. 
$$f(x) = \begin{cases} 2x+3, & x \ge 0\\ x+4, & x < 0 \end{cases}$$
 19. 
$$f(x) = \begin{cases} 3x-1, & x \ge -1\\ -5, & x < -1 \end{cases}$$
 20. 
$$f(x) = \begin{cases} -3x-1, & x \le 1\\ -5, & x > 1 \end{cases}$$



### Worksheet: Piecewise Functions

Evaluate the function for the given value of x.

$$f(x) = \begin{cases} 3, \text{ if } x \le 0\\ 2, \text{ if } x > 0 \end{cases} \qquad g(x) = \begin{cases} x + 5, \text{ if } x \le 3\\ 2x - 1, \text{ if } x > 3 \end{cases} \qquad h(x) = \begin{cases} \frac{1}{2}x - 4, \text{ if } x \le -2\\ 3 - 2x, \text{ if } x > -2 \end{cases}$$
  
1.  $f(2)$   
2.  $f(-4)$   
3.  $f(0)$   
4.  $f\left(\frac{1}{2}\right)$   
5.  $g(7)$   
6.  $g(0)$   
7.  $g(-1)$   
8.  $g(3)$   
9.  $h(-4)$   
10.  $h(-2)$   
11.  $h(-1)$   
12.  $h(6)$ 

Match the piecewise function with its graph.

Graph the function.

19.

20.

21.

 $f(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ -x + 1, & \text{if } 0 \le x \le 2 \\ x - 1, & \text{if } x > 2 \end{cases}$ f $f(x) = \begin{cases} x+3, & \text{if } x \le 0\\ 2x, & \text{if } x > 0 \end{cases}$ 

22. The admission rates at an amusement park are as follows.

Children 5 years old and under: free Children between 5 years and 12 years, inclusive: \$10.00 Children between 12 years and 18 years, inclusive: \$25.00 Adults: \$35.00

- a) Write a piecewise function that gives the admission price for a given age.
- b) Graph the function.

$$f(x) = \begin{cases} 2, & \text{if } x \le -3 \\ -1, & \text{if } -3 < x < 3 \\ 3, & \text{if } x \ge 3 \end{cases}$$