Honors Math 2 **Name: ________________**

Unit 7: Modeling Advanced Functions

- Model situations using inverse variation (**F-BF.1)**
- Explain why a solution is extraneous and give examples of extraneous solutions (**A-REI.2)**
- Create equations and inequalities in one variable (**A-CED.1)**
- Use equations and inequalities to solve problems. **(A-REI.2)**
- Represent constraints by equations or inequalities. (**A-CED.3)**

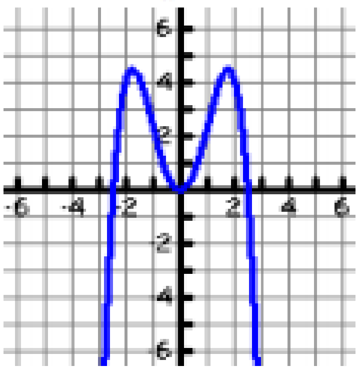
Tuesday 5/3	Even/Odd/Neither & Inverses
Wednesday 5/4	Solving Rational Equations part 1
Thursday 5/5	Review & Quiz
Friday 5/6	Solving Rational Equations part 2
Monday 5/9	Modeling Inverse Variation
Tuesday 5/10	Review
Wednesday 5/11	Test

Even, Odd or Neither Functions

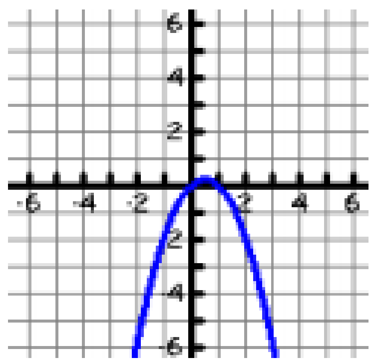
Even:	Odd:	Neither:
Symmetric about the ___ axis	Symmetric about the ___________	

Determine whether each of the following functions is *even*, *odd*, or *neither*. Justify your answer by stating how the graph is or isn't symmetrical.

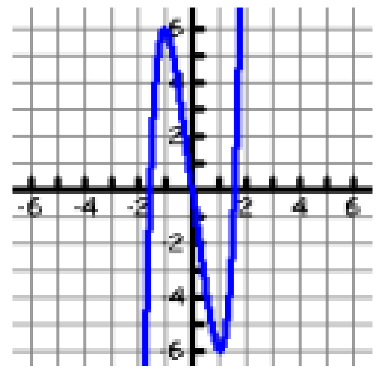
1. $f(x) = -\frac{1}{2}x^4 + 3x^2$



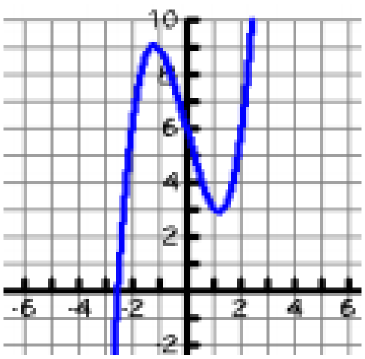
2. $g(x) = -x^2 + x$



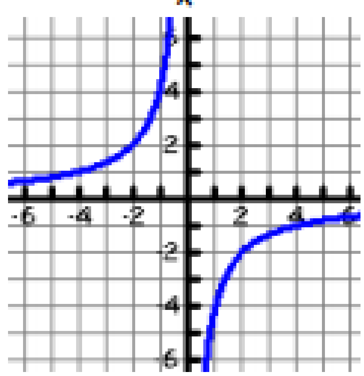
3. $h(x) = x^5 + x^3 - 8x$



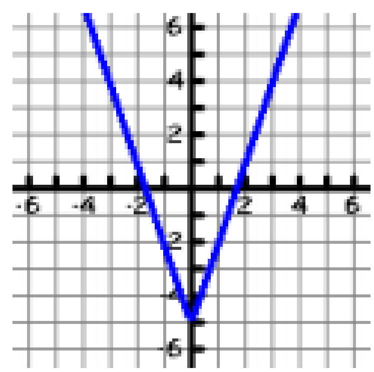
4. $f(x) = x^3 - 4x + 6$



5. $g(x) = \frac{-4}{x}$

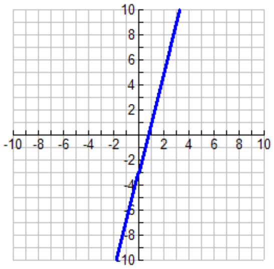


6. $h(x) = 3|x| - 5$

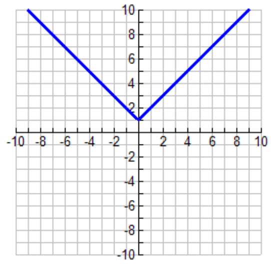


Determine using symmetry and algebraically whether the following functions are even, odd, or neither.

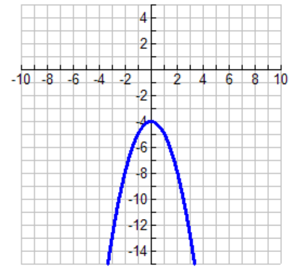
1. $f(x) = 4x - 3$



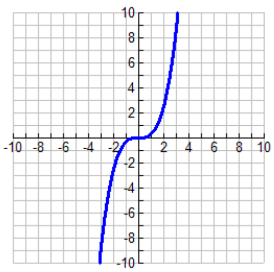
2. $f(x) = |x| + 1$



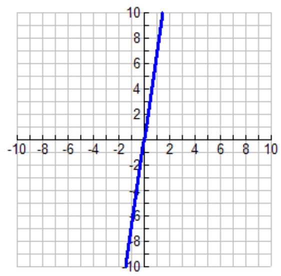
3. $f(x) = -x^2 - 4$



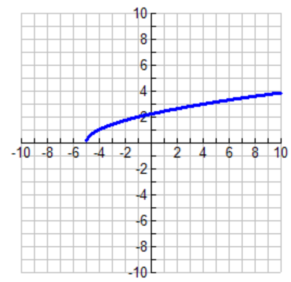
4. $f(x) = \frac{1}{3}x^3$



5. $f(x) = 7x$



6. $f(x) = \sqrt{x+5}$



7. $f(x) = 3x^2$

8. $f(x) = x^3 - 2$

9. $f(x) = 3x + 4$

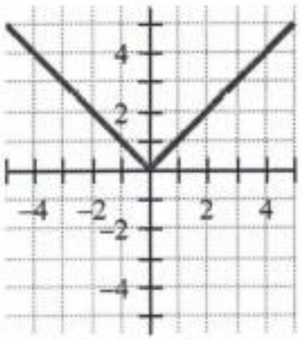
10. $f(x) = x^2 - 5$

11. $f(x) = 10x + 5$

12. $f(x) = 2(x+1)^3$

Determine whether each function graphed is even, odd, or neither

____3.



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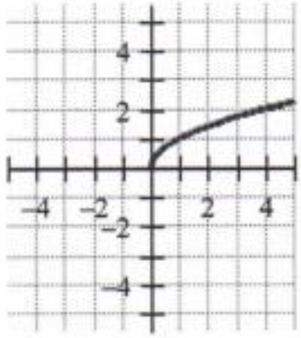

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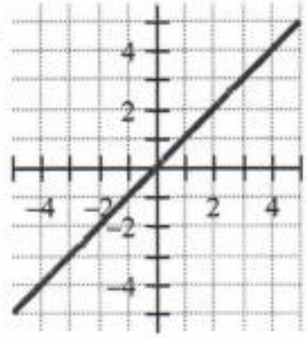

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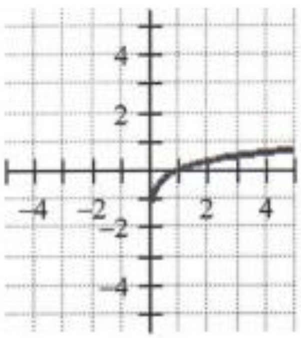

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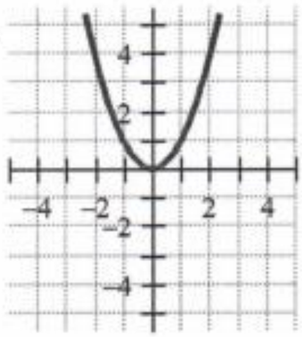

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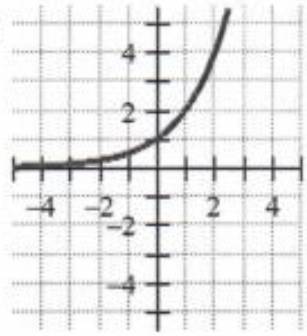

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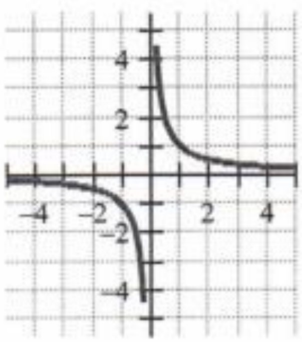

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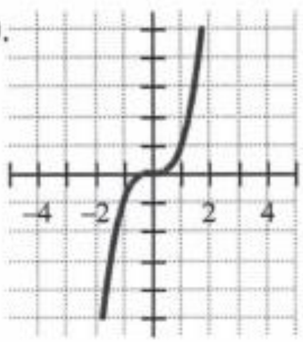

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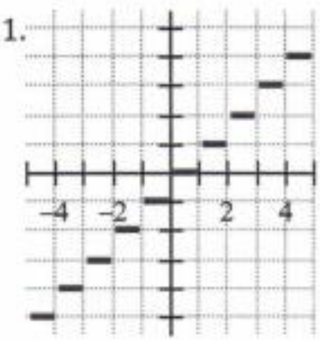


Determine algebraically whether each of the following functions is even, odd or neither.

12. $f(x)=4x+5$

13. $f(x)=x^3-x$

14. $f(x)=x^2-6$

15. $f(x)=x^3-x-2$

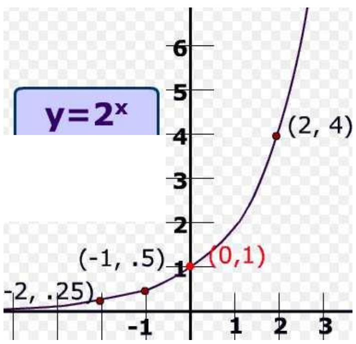
18. $f(x)=(x-4)^2$

19. $f(x)=x^4-x^2+4$

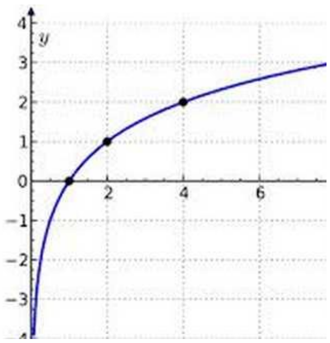
Inverse of a Function

Kathy and Kevin are sharing their graphs for the same set of data. Both students insist that they are correct, but yet their graphs are different. They have checked and re-checked their data and graphs. Can you explain what might have happened? Has this ever happened to you?

Kathy's Graph



Kevin's Graph



<u>**Vocabulary**</u>

Inverse of a Graph:

If you look at Kathy and Kevin's graph again, you might notice that the ordered pairs have been switched. For example in Kathy's graph the ordered pairs of (0,1) and (2,4) are (1,0) and (4,2) in Kevin's graph.

What familiar transformation (Unit 1A) do you see that happens with the inverse of a graph? ____________

INVESTIGATION: INVERSES

- **Which types of functions have inverses?**
- **How can rules for inverse functions be found?**

Consider the following functions:

1) f(x) = 6 + 3x 2) f(x) $= \frac{1}{3}(x-6)$

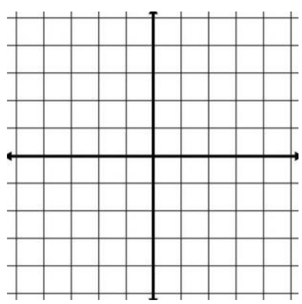
Part 1: Graphs of Inverse Functions

For each of the functions above, follow steps 1 – 4.

1) Make a table of 5 values and graph the function on graph paper.

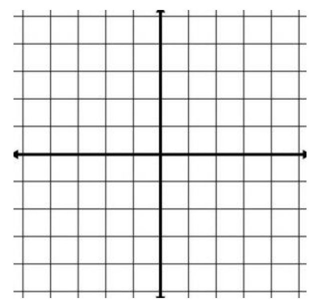
1) f(x) = 6 + 3x

X	Y



2) f(x) $= \frac{1}{3}(x-6)$

X	Y



2) Make another table by switching the x and y values and graph the inverse on the same coordinate plane.

1) f(x) = 6 + 3x

X	Y

2) f(x) $= \frac{1}{3}(x-6)$

X	Y

3) What do you notice about the two graphs?

4) What line are the inverses reflected over?

Part 2: Equations of Inverse Functions

We saw in part 1 of the investigation that functions 1 and 2 are inverse functions. We also know that we can find inverses of tables by switching the x and y values in a table. So the question we want to explore now is how to find the equation of an inverse function.

For each of the functions above, follow steps 5 – 6.

5) Take the function and switch the x and y values.
   1) 2)

6) Then solve for y.
   1) 2)

What do you notice about the two equations when you switch the x and y?

____

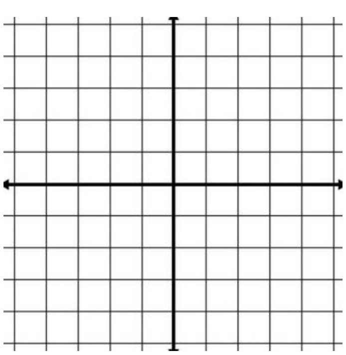
How do you know if an equation has an inverse?

If the graph is "one-to-one" then the equation has an inverse.

What does "one-to-one" mean? ________________________________

NOW TRY

Graph $y = x^2 + 3$ and find the inverse by interchanging the x and y values of several ordered pairs. Is the inverse a function? Check by graphing both $y = x^2 + 3$ and the inverse on the graph on the right.



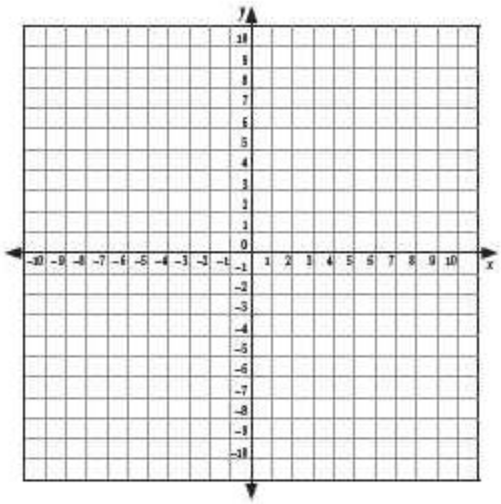
a) Switch x and y and solve for y

b) Is this an inverse? How do you know?

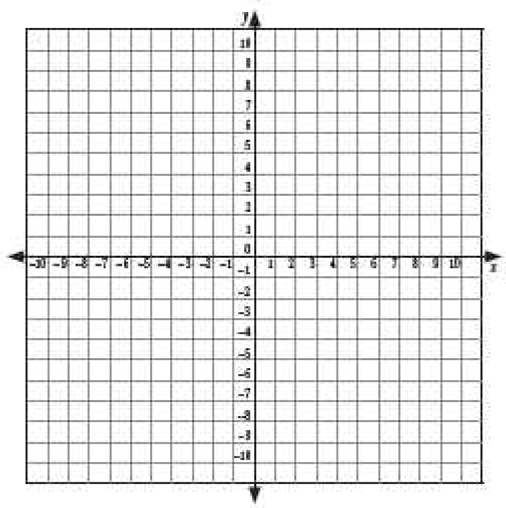
c) Is the graph one-to-one?

Practice: For each graph, a) state the inverse and b) graph the inverse and its function and c) determine if the function is one-to-one

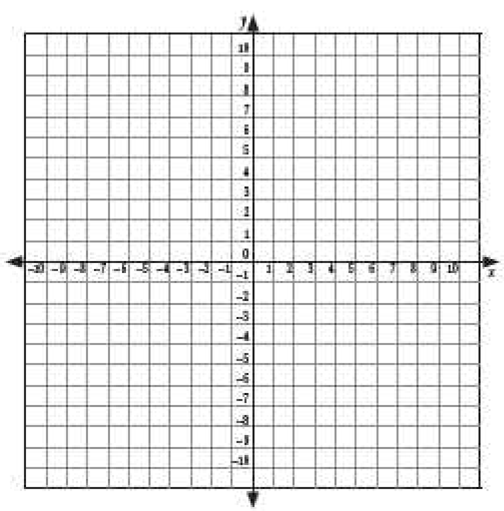
1. $f(x)=(x+3)^2$



2. $f(x)=x^3-4$



3. $f(x)=\dfrac{3}{x-2}$



Find the inverse of each function.

1) $g(x) = -\dfrac{1}{x-1} + 3$

2) $f(x) = x - 6$

3) $g(x) = -\dfrac{2}{5}x - 2$

4) $f(x) = \sqrt[5]{x+2} + 2$

5) $g(x) = \dfrac{-4 + \sqrt[3]{4x}}{2}$

6) $f(x) = \dfrac{4}{5}x - 4$

7) $h(n) = \dfrac{1}{n-2} - 2$

8) $g(x) = -\sqrt[5]{x} - 3$

9) $g(x) = -2x^5 - 2$

10) $f(x) = -\dfrac{1}{x} - 1$

A Lesson in Finding the LCD

LCD: least common denominator

Example: Find the LCD for the following pairs of expressions.

1) $\frac{5}{2}, \frac{7}{3}$

LCD:

2) $\frac{8}{3}, \frac{5}{6}$

LCD:

3) $\frac{5}{2}, 9$

LCD:

4) $\frac{3}{x}, \frac{5}{x^2}$

LCD:

5) $\frac{2}{3x}, \frac{4}{6x}$

LCD:

6) $\frac{1}{x-2}, \frac{1}{x-3}$

LCD:

We can use the LCD to get rid of fractions in equations by multiplying BOTH sides by the LCD.

Examples: Determine the LCD & multiply both sides by the LCD to get rid of the fractions. You don't actually have to solve.

1) $\frac{2}{5} + \frac{3}{10} = \frac{x}{5}$

2) $\frac{4}{x} + \frac{1}{x^2} = 5$

3) $\frac{2}{x} + 3 = \frac{9}{x}$

4) $\frac{x}{x-2} + \frac{4}{x-1} = 1$

5) $\frac{2}{x+2} + 4 = \frac{5}{x+2}$

Having trouble coming up with the LCD? Ask yourself: What can I multiply by to get rid of the denominators?

Solving Rational Equations

We learned how to graph inverse variation and the different aspects of the graph. In inverse variation, there are ______________________. We have a vertical asymptote because _____________________. We will begin solving rational equations, but we have to remember that there are some numbers that ________________________. These are called ________________________.

To find the excluded values, set the ______________ equal to zero and solve for the ______________________; the solutions are the excluded values. When solving rational equations, if all solutions of the rational equation are excluded values then there is ________________________ to the rational equation!

To solve simple rational equations, the cross product property can be utilized to eliminate the fraction leaving a linear equation to solve. **REMEMBER**: Check your final answers to make sure they are not an excluded value!

Examples: Using the cross product property, solve the following equations. Do not forget to determine the excluded values.

1. $\frac{6}{x} = \frac{3}{7}$ EV: ___________

2. $\frac{4}{x-7} = \frac{6}{x}$ EV: ___________

3. $\frac{-5}{x+4} = \frac{1}{x+4}$ EV: ___________

4. $\frac{6}{x+5} = \frac{x}{6}$ EV: ___________

Sometimes, you will have more than one rational expression on one or both sides of the equation. When this situation occurs the cross product property will not apply. You must find the ________________________ ____. By multiplying the entire equation by the LCD, the fractions are eliminated and you are left with an equation to simplify and solve.

Examples:The most efficient way of solving rational equations is by clearing the fractions using the LCD.

To solve $\frac{2x}{51} - 3 = \frac{5x}{17}$, we multiply both sides by the LCD __________.

Solve and check each answer.

1.) $\frac{4}{y} + \frac{1}{2} = \frac{5}{y}$

2.) $\frac{1}{z-1} + 5 = \frac{11}{z-1}$

We must be a little more careful when solving rational equations. The value for the variable that makes the denominator zero may not be a solution to the equation. We may end up with extraneous solutions.

3.) $\frac{12}{r^2-4} - \frac{3}{r-2} = \frac{5}{r+2}$

4.) $\frac{4}{3w} - \frac{1}{3} = w$

5.) $\frac{x+3}{x^2-x} - \frac{8}{x^2-1} = 0$

6.) $\frac{2w-5}{w^2-3w-4} = \frac{3w-1}{w^2+4w+3} - \frac{3}{w^2-w-12}$

7.) $\frac{4}{y-2} - \frac{1}{2-y} = \frac{25}{y+6}$

Examples: Multiply through by the LCD to solve the following equations. Do not forget to determine the excluded values.

5. $\frac{2}{x} - 3 = \frac{8}{x}$ EV: __________

6. $\frac{7x}{x-3} + 4 = \frac{x+1}{x-3}$ EV: __________

Practice: For the following problems, solve each rational equation.

1. $\frac{x+3}{2x} = \frac{5}{8}$

2. $\frac{1}{3z} + \frac{1}{8} = \frac{4}{3z}$

3. $\frac{y+3}{y-1} = \frac{y+2}{y-3}$

4. $\frac{x+1}{x-1} + \frac{2}{x} = \frac{x}{x+1}$

5. $\frac{1}{6} - \frac{1}{x} = \frac{4}{3x^2}$

6. $\frac{2x+3}{x-1} - \frac{2x-3}{x+1} = \frac{10}{x^2-1}$

7. $\frac{b}{b+3} - \frac{b}{b-2} = \frac{10}{b^2+b-6}$

8. $\frac{3}{x+2} - \frac{4}{x-2} = 5$

9. $\frac{5}{x} = 1 + \frac{3}{x+2}$

Solving Rational Equations Worksheet

Solve and find the excluded values for each equation.

1. $\frac{x}{5}=\frac{7}{3}$

2. $\frac{2x}{16}=\frac{2}{x}$

3. $\frac{4}{x-5}=\frac{2}{x+8}$

4. $\frac{1}{4}+\frac{4}{y}=\frac{1}{y}$

5. $\frac{1}{5}-\frac{2}{5a}=\frac{1}{a}$

6. $\frac{h}{9}-\frac{8}{h}=\frac{1}{9}$

7. $\frac{m}{m+9}=\frac{9}{m+9}+2$

8. $\frac{w}{w+4}+\frac{1}{3}=\frac{-12}{w+4}$

Summary of Direct and Inverse Variation

	DIRECT VARIATION	INVERSE VARIATION
What is it?	A set of ordered pairs with a constant ____________!	A set of ordered pairs with a constant ____________!
Equation Form		
Examples	• • •	• • •
How do you test for it?		
Graph		
How do you solve for missing values?		

Direct vs. Inverse Variation

Determine if the given ordered pairs, tables or graphs are direct variation, inverse variation or neither. If it is direct variation or inverse variation, write the equation.

1. $y = \frac{36}{x}$

☐ DIRECT ☐ INVERSE ☐ NEITHER

2. $y = -\frac{2}{5}x + 1$

☐ DIRECT ☐ INVERSE ☐ NEITHER

3. $4y = x$

☐ DIRECT ☐ INVERSE ☐ NEITHER

4. $xy = -18$

☐ DIRECT ☐ INVERSE ☐ NEITHER

5. $x - 3y = 15$

☐ DIRECT ☐ INVERSE ☐ NEITHER

6. $\frac{y}{2} = x$

☐ DIRECT ☐ INVERSE ☐ NEITHER

7. $x = \frac{16}{y}$

☐ DIRECT ☐ INVERSE ☐ NEITHER

8. $x + y = 0$

☐ DIRECT ☐ INVERSE ☐ NEITHER

9.

x	y
1.5	20
2.5	12
4	7.5
5	6

☐ DIRECT ☐ INVERSE ☐ NEITHER

10.

x	y
31	217
20	140
17	119
12	84

☐ DIRECT ☐ INVERSE ☐ NEITHER

11.

x	3	7	5	16
y	36	105	50	48

☐ DIRECT ☐ INVERSE ☐ NEITHER

12.

x	4	5	1.6	20
y	16	12.8	40	3.2

☐ DIRECT ☐ INVERSE ☐ NEITHER

13.

x	2	4	8	11
y	1.5	3	6	8.25

☐ DIRECT ☐ INVERSE ☐ NEITHER

14.

x	0.5	1	3	6
y	6	3	1	0.5

☐ DIRECT ☐ INVERSE ☐ NEITHER

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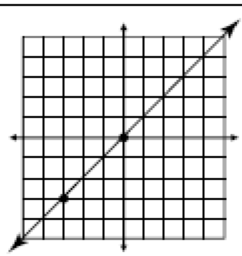

☐ DIRECT ☐ INVERSE ☐ NEITHER

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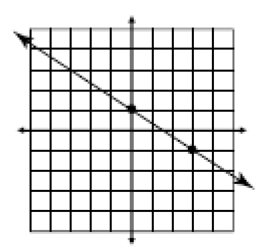

☐ DIRECT ☐ INVERSE ☐ NEITHER

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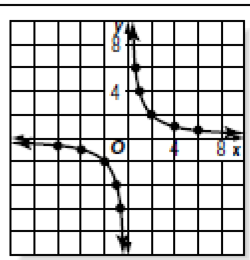

☐ DIRECT ☐ INVERSE ☐ NEITHER

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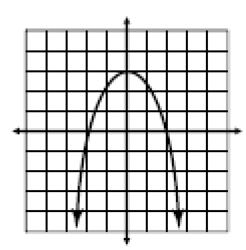

☐ DIRECT ☐ INVERSE ☐ NEITHER

Applications of Direct and Inverse Variation

WHEN DO WE USE DIRECT?	WHEN DO WE USE INVERSE?
In situations where as one variable _____, the other variable _____.	In situations where as one variable _____, the other variable _____.

1. The sales at a baseball game vary directly with the number of people attending. If the sales are \$12,000 when 800 people attend, determine how many people attend if the sales for a game are \$15,000.

2. Time traveling varies inversely with average speed. If a train travels between two cities in 3 hours at an average speed of 65 mph, how long would it take at an average speed of 78 mph?

3. Ounces of medication vary directly with the weight of the patient. If a 120 lb. adult requires three-fourths of an ounce of medication, how many ounces are required for a 200 lb. adult?

4. A company that produces laptops has determined that the number o f laptops it can sell varies inversely to the price of the laptop. Two thousand laptops can be sold when the price is \$2,500. How many laptops can be sold if the price of the laptop is \$1,000?

5. The number of shares varies directly with the amount of dividends received per year. If Kim owns 5 shares of stock and receives $12 per year, how many shares of stock will she need to receive $24?

6. The width of a rectangle varies inversely with its length. If the width is 4 when the length is 18, find the width when the length is 8.

7. The gallons of water used to take a shower vary directly with the number of minutes in the shower. If an 8 minute shower uses 48 gallons of water, find the constant of variation.

8. The cost per person to rent a cottage in the Outer Banks is inversely proportional to the number of people who share the rent. If 7 people share the rent and it costs $42 per person, find the constant of variation.

9. A marathon is roughly 26.2 miles long. Which equation could be used to determine the time, t, it takes to run a marathon as a function of the average speed, s, of the runner where t is in hours and s is in miles.

a. $t = 26.2 - 26.2s$ b. $t = 26.2 - \frac{s}{26.2}$

c. $t = 26.2s$ d. $t = \frac{26.2}{s}$

10. The time t in hours that it takes x people to plant n trees varies directly with the number of trees, and inversely with the number of people. Suppose 6 people can plant 12 trees in 3 hour. How many people are needed to plant 28 trees in 5 hours and 15 minutes?

a. 6 b. 7 c. 8 d. 9

11. The force, F, acting on a charged object varies inversely to the square of its distance, r, from another charged object. When the two objects are 0.64 meter apart, the force acting on them is 8.2 Newtons. Approximately how much force would the object feel if it is at a distance of 0.77 meter from the other object?

a. 1.7 Newtons b. 5.7 Newtons

c. 11.9 Newtons d. 12.9 Newtons

12. The pressure P of a compressed gas is inversely proportional to the volume V. If there is a pressure of 25 pounds per square inch when the volume of gas is 400 cubic inches, find the pressure when the gas is compressed to 200 cubic inches.

a. $200P = 10000$ b. $25P = 80000$

c. $400P = 5000$ d. none of the above

13. The time it takes to complete a job varies inversely as the number of people working. It took 4 hours for 7 electricians to wire a building. How long will it take 3 electricians to finish the job?

a. 1 hr 43 min b. 5 hr 15 min

c. 7 hr 30 min d. 9 hr 20 min

14. On a ski lift, the distance between chairs is inversely proportional to the number of chairs. At a ski resort, one lift has 80 chairs space 16m apart. What is the constant of variation?

a. 1280 b. 5 c. 1/5 d. 1/1280

VARIATION PRACTICE WORKSHEET - HONORS

1. The frequency of a radio signal varies inversely as the wave length. A signal of frequency 1200 kilohertz (kHz), which might be the frequency of an AM radio station, has wave length 250m. What frequency has a signal of wave length 400m?

2. By Ohm's law, the current flowing in a wire is inversely proportional to the resistance of the wire. If the current is 5 amperes (A) when the resistance is 24 ohms (Ω), for what resistance will the current be 8 A?

3. The conductance of a wire varies directly as the square of the wire's diameter and inversely as its length. Fifty meters of wire with diameter 2 mm has conductance 0.12 mho (mho, which is "ohm" spelled backwards, is a unit of conductance). If a wire of the same material has length 75 m and diameter 2.5 mm, what is its conductance?

In 4 – 5, use the fact that the intensity of light, measured in lux, is inversely proportional to the square of the distance between the light source and the object illuminated.

4. A light meter 7.5 m from a light source registers 24 lux. What intensity would it register 15 m from the light source?

5. A light hangs 4.8 ft above the center of a circular table 7.2 ft in diameter. If the illumination is 24 lux at the center of the table, what is it at the edge of the table?

6. The stretch in a wire under a given tension varies directly as the length of the wire and inversely as the square of its diameter. A wire having length 2 m and diameter 1.5 mm stretches 1.2 mm. If a second wire of the same material (and under the same tension) has length 3 m and diameter 2.0 mm, find the amount of stretch.

7. If the sales tax on a \$60 purchase is \$3.90, what would it be on a \$280 purchase?

8. On a certain map, a field 280 ft long is represented by a 5 in by 8 in rectangle. How wide is the field?

9. A public opinion poll found that of a sample of 450 voters, 252 favored a school bond measure. If 20,000 people vote, about how many are likely to vote for the bond measure?

10. The speed of an object falling from rest in a vacuum is directly proportional to the time it has fallen. After an object has fallen for 1.5 s, its speed is 14.7 m/s. What is its speed after it has fallen 5 s?

11. Gas Law Problem: In chemistry you learn Boyle's Law, which states that the volume of a fixed amount of gas (at constant temperature) is inversely proportional to the pressure of the gas.
    a. Write the particular equation expressing volume in terms of pressure if a pressure of 46 pounds per square inch (psi) compresses the gas to a volume of 360 cubic feet.
    b. What pressure would be necessary to compress the gas to a volume of 276 cubic feet?
    c. According to your model, could you compress the gas to zero volume? How do you tell?
    d. Show that the product of the pressure and the volume is constant. (This is the way Boyle's Law is stated in some chemistry books.)
    e. Show that if (P_1, V_1) and (P_2, V_2) are ordered pairs of pressure and volume, then $P_1V_1 = P_2V_2$. (This is the way Boyle's Law is stated in many chemistry books.)