

I. Find the LCD of each rational equation and build up each fraction so that they all have the same denominator.

1. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$

2. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$
 $(y-5)(y+2)$

3. $\frac{p+10}{p^2-p} = \frac{4}{p}$
 $p(p-1)$

LCD: (s+2)

LCD: (y-5)(y+2)

LCD: p(p-1)

II. Solve each of the following rational equations. Make sure to find any excluded values.

4. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$

5. $\frac{1}{n-2} + \frac{1}{n+2} = \frac{3}{n^2-4}$
 $(n-2)(n+2)$

LCD: (x-1)(x-2)(x+1) EV: x = 1, 2, -1

LCD: (n-2)(n+2) EV: n = 2, -2

Solution: no solution (see separate sheet)

Solution: n = 3/2 (see separate sheet)

6. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$

7. $\frac{4}{w-2} = \frac{-1}{w+3}$

LCD: 2h(h-1) EV: h = 0, 1

LCD: (w-2)(w+3) EV: w = 2, -3

Solution: h = 11/5 (see separate sheet)

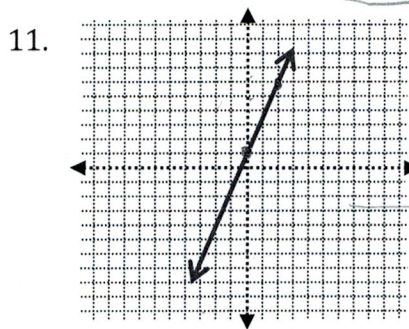
Solution: w = -2 (see separate sheet)

III. Find the inverse of the following functions. (see separate sheet)

8. $f(x) = -5x - 11$ $y = -\frac{1}{5}(x+11)$

9. $f(x) = (x-2)^2$ $y = \sqrt{x} + 2$

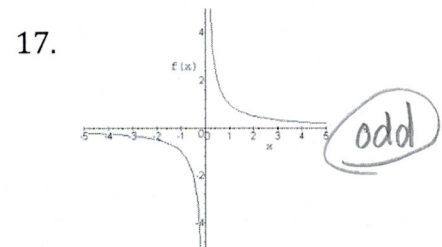
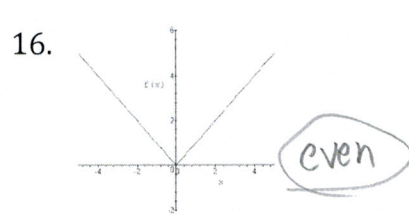
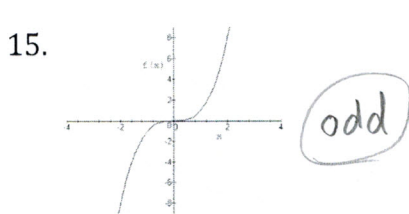
10. $f(x) = \frac{1}{3}x + 7$ $y = 3x - 21$



12. $f(x) = \sqrt[3]{x-7} + 2$
 $y = (x-2)^3 + 7$

13. $f(x) = (x-2)^5 - 1$
 $y = \sqrt[5]{x+1} + 2$

IV. Determine if the following functions are even, odd or neither. Explain.



18. $f(x) = 4x - 9$
 \uparrow odd \uparrow even
neither

19. $f(x) = 3x^7 - 2x^5 + 3x^3$
 \uparrow odd \uparrow odd \uparrow odd
odd

20. $f(x) = 4x^6 - 7x^4 + 9$
 \uparrow even \uparrow even \uparrow even
even

$$\textcircled{4} \quad \frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$$

$$6(x-2)(x+1) = 4(x-1)(x+1) + 2(x-1)(x-2)$$

$$6(x^2 - x - 2) = 4(x^2 - 1) + 2(x^2 - 3x + 2)$$

$$6x^2 - 6x - 12 = 4x^2 - 4 + 2x^2 - 6x + 4$$

$$\cancel{6x^2} - \cancel{6x} - 12 = \cancel{6x^2} - \cancel{6x}$$

$$-12 = 0$$

no solution

$$\textcircled{9} \quad f(x) = (x-2)^2$$

$$y = (x-2)^2$$

$$x = (y-2)^2$$

$$x^{1/2} = y-2$$

$$x^{1/2} + 2 = y$$

$$y = \sqrt{x} + 2$$

$$\textcircled{5} \quad \frac{1}{n-2} + \frac{1}{n+2} = \frac{3}{n^2-4}$$

$$n+2 + n-2 = 3$$

$$2n = 3$$

$$n = \frac{3}{2}$$

$$f(x) = \frac{1}{3}x + 7 \quad \textcircled{10}$$

$$y = \frac{1}{3}x + 7$$

$$x = \frac{1}{3}y + 7$$

$$x-7 = \frac{1}{3}y$$

$$y = 3x - 21$$

$$\textcircled{11} \quad y = \frac{5}{2}x + 1$$

$$x = \frac{5}{2}y + 1$$

$$x-1 = \frac{5}{2}y$$

$$y = \frac{2}{5}(x-1)$$

$$\textcircled{6} \quad \frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$$

$$h-1 + 10(h-1) = 3(2h)$$

$$h-1 + 10h-10 = 6h$$

$$11h-11 = 6h$$

$$5h = 11$$

$$h = \frac{11}{5}$$

$$\textcircled{12} \quad f(x) = \sqrt[3]{x-7} + 2$$

$$y = \sqrt[3]{x-7} + 2$$

$$x = \sqrt[3]{y-7} + 2$$

$$x-2 = \sqrt[3]{y-7}$$

$$(x-2)^3 = y-7$$

$$y = (x-2)^3 + 7$$

$$\textcircled{13} \quad f(x) = (x-2)^5 - 1$$

$$y = (x-2)^5 - 1$$

$$x = (y-2)^5 - 1$$

$$x+1 = (y-2)^5$$

$$y-2 = \sqrt[5]{x+1}$$

$$y = \sqrt[5]{x+1} + 2$$

$$\textcircled{7} \quad \frac{4}{w-2} = \frac{-1}{w+3}$$

$$4w + 12 = -w + 2$$

$$5w = -10$$

$$w = -2$$

$$\textcircled{8} \quad f(x) = -5x - 11$$

$$y = -5x - 11$$

$$x = -5y - 11$$

$$x+11 = -5y$$

$$y = -\frac{1}{5}(x+11)$$

V. Direct, Inverse and Joint Variation

21. If $x=5$ when $y=20$, find y when $x=10$ if x and y vary directly.

$$y = kx \quad 20 = 5k \quad y = 4x$$

$$k = 4 \quad y = 4(10) \quad \boxed{y = 40}$$

22. Suppose y varies inversely with the square of x , and $y=50$ when $x=4$. Find y when $x=5$.

$$y = \frac{k}{x^2} \quad 50 = \frac{k}{4^2} \quad y = \frac{800}{x^2} \quad y = \frac{800}{5^2} = \frac{800}{25} = \boxed{32}$$

$$k = 800$$

23. Suppose that y varies directly with x and inversely with the square of z . If $x=48$ when $y=8$ and $z=3$. Find the constant of variation (k) and x when $y=12$ and $z=2$.

$$y = \frac{kx}{z^2} \quad 8 = \frac{k(48)}{3^2} \quad \boxed{k = \frac{3}{2} \text{ or } 1.5} \quad 12 = \frac{1.5x}{2^2} \quad \boxed{x = 32}$$

$$72 = 48k \quad 48 = 1.5x$$

24. When a person swims underwater, the pressure in his or her ears varies directly with the depth at which he or she is swimming. At 10 feet, the pressure is about 4.3 pounds per square inch. Find the pressure if the depth is 60 feet.

$$P = kD \quad P = 0.43(60)$$

$$4.3 = 10k \quad \boxed{P = 25.8 \text{ pounds per square inch}}$$

$$k = 0.43$$

25. When air is pumped into a tire, the pressure required varies inversely as the volume of the air. If the pressure is 30 lb/in² when the volume is 140 in³, find the pressure when the volume is 100 in³.

$$P = \frac{k}{V} \quad 30 = \frac{k}{140} \quad P = \frac{4200}{V} \quad P = \frac{4200}{100}$$

$$k = 4200 \quad \boxed{P = 42 \text{ lb/in}^2}$$

26. Determine if the following tables are direct variation, inverse variation or neither. Then determine the constant of variation.

a.

| x | y |
|----|----|
| -4 | 6 |
| -2 | 12 |
| 8 | -3 |

$\frac{6}{-4} \neq \frac{12}{-2}$
not direct

$-4 \cdot 6 = -24$
 $-2 \cdot 12 = -24$
 $8 \cdot -3 = -24$

$\boxed{\text{inverse variation } k = -24}$

b. (6, 4.5)(8,6)(10,7.5)

$\frac{4.5}{6} = \frac{6}{8} = \frac{7.5}{10}$ direct variation

$k = \frac{4.5}{6} = \frac{3}{4} \text{ or } .75$

c.

| x | y |
|----|----|
| 1 | 3 |
| 4 | 9 |
| 10 | 21 |

$\frac{3}{1} \neq \frac{9}{4}$
not direct

$1 \cdot 3 = 3$
 $4 \cdot 9 = 36$
not inverse

$\boxed{\text{neither}}$

27. If the point (12, 4) lies on the graph of a direct variation, what is the constant of variation?

$$y = kx \quad 4 = 12k$$

$$\boxed{k = \frac{4}{12} = \frac{1}{3}}$$

28. If y is inversely proportional to x and $y=7$ when $x=9$, find x when $y=14$.

$$y = \frac{k}{x} \quad 7 = \frac{k}{9} \quad y = \frac{63}{x} \quad 14 = \frac{63}{x}$$

$$k = 63 \quad \boxed{x = \frac{63}{14} = \frac{9}{2} = 4.5}$$

29. Which set of ordered pairs satisfies an inverse variation?

- a. (6,3) and (8,4) b. (2,3) and (4,5) c. (4,-2) and (-5, 10) d. (2,6) and (-3, -4)
- $6 \cdot 3 \neq 8 \cdot 4$ $2 \cdot 3 \neq 4 \cdot 5$ $4 \cdot -2 \neq -5 \cdot 10$ $2 \cdot 6 = -3 \cdot -4$

30. The time required to complete a job varies inversely as the number of people working. It took 4 hours for 7 electricians to wire a building. How long would it have taken 3 electricians to have done the job?

$$t = \frac{k}{n} \quad k = 28$$

$$4 = \frac{k}{7} \quad t = \frac{28}{n}$$

$$\boxed{t = \frac{28}{3} = 9\frac{1}{3} = 9 \text{ hours } 20 \text{ min}}$$